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CHAPTER WISE TOPIC WISE

LINE BY LINE QUESTIONS

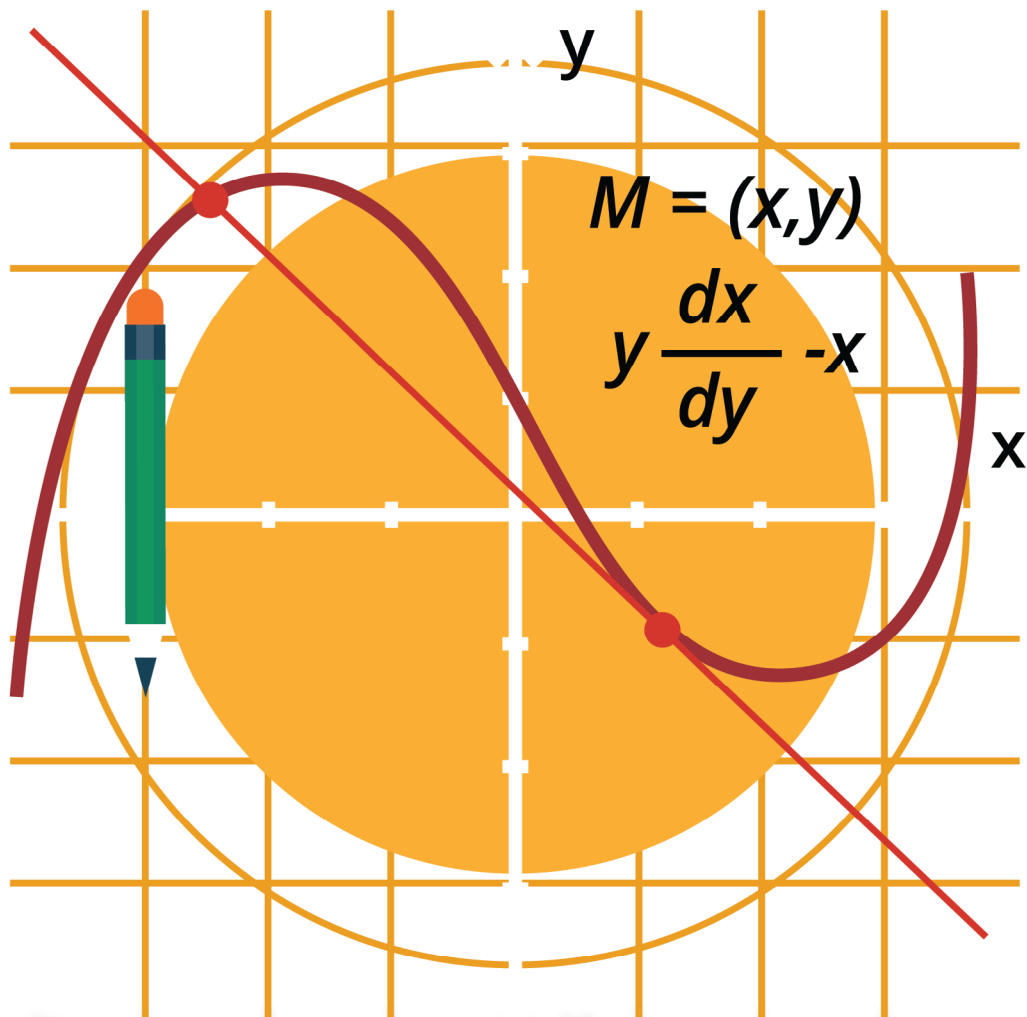
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BY
SCHOOL OF
EDUCATORS

Mathematics

Class 12





DETERMINANTS & MATRICES

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DETERMINANTS

1. INTRODUCTION TO DETERMINANTS

1.1 Definition

- (i) The determinant consisting two rows and two columns is

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}. \text{ Its value is given by :}$$

$$D = a_1 b_2 - a_2 b_1$$

- (ii) A determinant which consists of 3 rows and 3 columns is called a 3rd-order-determinant and is of the following form.

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

It's value is :

$$D = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{12}a_{21}a_{33}$$

1.2 Minors and Cofactors

$$\text{Let } \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Here a_{ij} = Element in i^{th} row and j^{th} column of Δ .

Minor of a_{ij} :

It is defined as the value of the determinant obtained by eliminating the i^{th} row and j^{th} column of Δ .

We denote the minor of a_{ij} by M_{ij} .

$$\text{e.g. } M_{11} = \text{minor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{32}a_{23}$$

Cofactor of a_{ij} :

Denoted by C_{ij}

Cofactor of a_{ij} (C_{ij}) = $(-1)^{i+j}$ Minor of a_{ij}

$$\text{e.g. Cofactor of } a_{11} (C_{11}) = (-1)^{1+1} M_{11} = M_{11}$$

1.3 Evaluation of Determinant

Value of any determinant can be obtain by adding product of all elements of a row (or column) to their corresponding cofactors.

$$\text{e.g. } \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{aligned} \Delta &= a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} \\ &= a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23} \\ &= a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33} \\ &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= a_{12}C_{12} + a_{22}C_{22} + a_{32}C_{32} \\ &= a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33} \end{aligned}$$

2. PROPERTIES OF DETERMINANTS

- (i) The value of a determinant remains unaltered; if the rows and columns are interchanged,

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

- (ii) If any two rows (or columns) of a determinant be interchanged, the value of determinant changes in sign only.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and } D' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D' = -D.$$

- (iii) If a determinant has all the elements zero in any row (or column) then its values is zero.

$$D = \begin{vmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$



- (iv) If a determinant has any two rows (or columns) identical or proportional, then its value is zero.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ ka_1 & kb_1 & kc_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

- (v) If all the elements of any row (or column) are multiplied by the same number, then the determinant is multiplied by that number.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}; \text{ and } D' = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\text{Then } D' = KD$$

- (vi) If each element of any row (or column) can be expressed as a sum of two terms then the determinant can be expressed as the sum of two determinants, i.e.

$$\begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

- (vii) The value of determinant is not altered by adding to the elements of any row (or column) a constant multiple of the corresponding elements of any other row (or column).

e.g.

$$R_1 \rightarrow R_1 + mR_2 \text{ (change } R_1 \text{ as sum of } R_1 \text{ and } m(R_2)).$$

$$R_3 \rightarrow R_3 + nR_2 \text{ (change } R_3 \text{ as sum of } R_3 \text{ and } n(R_2)).$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and}$$

$$D' = \begin{vmatrix} a_1 + ma_2 & b_1 + mb_2 & c_1 + mc_2 \\ a_2 & b_2 & c_2 \\ a_3 + na_2 & b_3 + nb_2 & c_3 + nc_2 \end{vmatrix}$$

$$\text{Then } D' = D.$$

3. CRAMER'S RULE

(Not in CBSE Syllabus)

- (i) Two Variables

$$\text{If } a_1x + b_1y = c_1 \quad \dots (i)$$

$$a_2x + b_2y = c_2 \quad \dots (ii)$$

$$\text{then } x = \frac{D_x}{D}, y = \frac{D_y}{D}$$

values of x, y are unique, if $D \neq 0$. where, $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$,

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \quad D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

Similarly 'n' equations in 'n' variables can be solved.

- (ii) Three Variables

$$\text{Let, } a_1x + b_1y + c_1z = d_1 \dots\dots\dots(i)$$

$$a_2x + b_2y + c_2z = d_2 \dots\dots\dots(ii)$$

$$a_3x + b_3y + c_3z = d_3 \dots\dots\dots(iii)$$

$$\text{Then, } x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{D_z}{D}$$

$$\text{Where, } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}; D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix};$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \text{ and } D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Consistency of a System of Equations

- (i) If $D \neq 0$ then the given system of equations are consistent and have unique solution.
- (ii) If $D = 0$ but at least one of D_x, D_y, D_z is not zero then the equations are inconsistent and have no solution.
- (iii) If $D = D_x = D_y = D_z = 0$ then the given system of equations are consistent and have infinite solution except the case of parallel planes when there is no solution.
- (iv) If $d_1 = d_2 = d_3 = 0$ then system of equation is called Homogenous system of equations.



- (v) Solution of Homogenous Equations is always consistent, as $x = 0 = y = z$ is always a solution. This is known as TRIVIAL solution.
- (vi) For Homogenous Equations, if $D \neq 0$. Then $x = 0 = y = z$ is the only solution.
- (vii) For Homogenous Equations, if $D = 0$, then there exists non zero solutions [NON TRIVIAL SOLUTIONS] also.

4. APPLICATION OF DETERMINANTS

Following examples of short hand writing of large expressions are:

- (i) Area of triangle whose vertices are $(x_r, y_r); r = 1, 2, 3$, is:

$$D = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If $D = 0$ then the three points are collinear.

- (ii) Equation of straight line passing through (x_1, y_1) &

$$(x_2, y_2) \text{ is } \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

- (iii) The lines :

$$a_1x + b_1y + c_1 = 0 \dots\dots\dots(1)$$

$$a_2x + b_2y + c_2 = 0 \dots\dots\dots(2)$$

$$a_3x + b_3y + c_3 = 0 \dots\dots\dots(3)$$

are concurrent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

This is condition for the consistency of simultaneous linear equation in two variables.

- (iv) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$$abc + 2fgh - af^2 - bg^2 - ch^2 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

- (v) To find the variable (x, y, z etc) in linear equations (Cramer's rule)

5. SOME MORE PROPERTIES OF DETERMINANT

- (i) Determinant of a skew-symmetric matrix of odd order is zero.

e.g. $D = \begin{vmatrix} 0 & 2 & 9 \\ -2 & 0 & \log_a b \\ -9 & \log_a \left(\frac{1}{b}\right) & 0 \end{vmatrix} = 0$

- (ii) Determinant of a skew-symmetric matrix of even order is always a perfect square.

e.g. $D = \begin{vmatrix} 0 & 5 \\ -5 & 0 \end{vmatrix}_{2 \times 2} = 25$

- (iii) Δ^{n-1} , where n is order of the determinant is equal to the determinant made from cofactors of elements of Δ .

e.g. $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}_{3 \times 3}^{3-1} = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$

Where A_i 's are co-factors of a_i 's

- (iv) Determinant of a diagonal matrix is product of its diagonal elements

e.g. $D = \begin{vmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{vmatrix} = 5 \times 2 \times 6 = 60$

- (v) If a determinant considered as a polynomial becomes zero when $x = a$, then $x - a$ is factor of this. (This is an application of Factor Theorem)

$$D = \begin{vmatrix} x & a & a^2 \\ a & x & x^2 \\ a & x & a \end{vmatrix}$$

Because $D = 0$ when $x = a$ so $x - a$ is factor of D .

- (vi) The sum of the products of the elements of the i^{th} row/column with the co-factor of the corresponding elements of k^{th} row/column is zero provided $i \neq k$.



i.e. (i) $\sum_{j=1}^n a_{ij}C_{kj}=0$; if $i \neq k$

(ii) $\sum_{i=1}^n a_{ij}C_{ik}=0$ if $j \neq k$

(vii) $|AB| = |A||B|$

i.e. $A = \begin{vmatrix} 5 & -2 \\ -1 & -1 \end{vmatrix}$; $B = \begin{vmatrix} -2 & -3 \\ -4 & 1 \end{vmatrix}$; $AB = \begin{vmatrix} -2 & -17 \\ 6 & 2 \end{vmatrix}$

$|A| = -7$ $|B| = -14$

$|A||B| = -7 \times -14 = 98$,

$|AB| = -4 + 102 \Rightarrow |AB| = 98$.

(viii) Determinant of a triangular matrix is product of its diagonal elements only.

$D = \begin{vmatrix} 3 & 2 & 1 \\ 0 & 4 & 3 \\ 0 & 0 & -1 \end{vmatrix} = 3 \times 4 \times -1 = -12$

$D' = \begin{vmatrix} 5 & 0 & 0 \\ 4 & 9 & 0 \\ 3 & 2 & 1 \end{vmatrix} = 5 \times 9 \times 1 = 45$

(iii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

(iv) $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

7. MULTIPLICATION OF TWO DETERMINANTS

(a) $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} \ell_1 & m_1 \\ \ell_2 & m_2 \end{vmatrix} = \begin{vmatrix} a_1\ell_1 + b_1\ell_2 & a_1m_1 + b_1m_2 \\ a_2\ell_1 + b_2\ell_2 & a_2m_1 + b_2m_2 \end{vmatrix}$

(b) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \\ \ell_3 & m_3 & n_3 \end{vmatrix}$

$= \begin{vmatrix} a_1\ell_1 + b_1\ell_2 + c_1\ell_3 & a_1m_1 + b_1m_2 + c_1m_3 & a_1n_1 + b_1n_2 + c_1n_3 \\ a_2\ell_1 + b_2\ell_2 + c_2\ell_3 & a_2m_1 + b_2m_2 + c_2m_3 & a_2n_1 + b_2n_2 + c_2n_3 \\ a_3\ell_1 + b_3\ell_2 + c_3\ell_3 & a_3m_1 + b_3m_2 + c_3m_3 & a_3n_1 + b_3n_2 + c_3n_3 \end{vmatrix}$

6. SPECIAL DETERMINANTS

(i) Circulant Determinants :

The elements of the rows (or columns) are in cyclic arrangement

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$.

$= -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

8. SUMMATION OF DETERMINANTS

Let $\Delta_r = \begin{vmatrix} f(r) & a & \ell \\ g(r) & b & m \\ h(r) & c & n \end{vmatrix}$

Where a, b, c, ℓ, m and n are constants independent of r , then

$\sum_{r=1}^n \Delta_r = \begin{vmatrix} \sum_{r=1}^n f(r) & a & \ell \\ \sum_{r=1}^n g(r) & b & m \\ \sum_{r=1}^n h(r) & c & n \end{vmatrix}$



Here functions of r can be the elements of only one row or column. None of the elements other than row or column should be dependent on r .

9. DIFFERENTIATION AND INTEGRATION OF DETERMINANT

$$\Delta(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$$

Then

$$\Delta'(x) = \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1'(x) & g_2'(x) & g_3'(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1'(x) & h_2'(x) & h_3'(x) \end{vmatrix}$$

$$+ \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2'(x) & h_3'(x) \end{vmatrix}$$

Integration of determinant

$$\text{If } \Delta(x) = \begin{vmatrix} f(x) & g(x) \\ \lambda_1 & \lambda_2 \end{vmatrix},$$

$$\text{then } \int_a^b \Delta(x) dx = \begin{vmatrix} \int_a^b f(x) dx & \int_a^b g(x) dx \\ \lambda_1 & \lambda_2 \end{vmatrix}.$$

Here $f(x)$ and $g(x)$ are functions of x and λ_1, λ_2 are constants.

NOTES:

This formula is only applicable if there is a variable only in one row or column, otherwise expand the determinant and then integrate.

$$\text{Example : If } f(x) = \begin{vmatrix} x^3 & \cos^2 x & 2x^4 \\ \tan^5 x & 1 & \sec 2x \\ \sin^3 x & x^4 & 5 \end{vmatrix} \text{ then } \int_{-\pi/2}^{\pi/2} f(x) dx =$$

- (a) 2 (b) -2 (c) 0 (d) none of these

Sol. We have

$$f(-x) = \begin{vmatrix} -x^3 & \cos^2 x & 2x^4 \\ -\tan^5 x & 1 & \sec 2x \\ -\sin^3 x & x^4 & 5 \end{vmatrix} = -f(x)$$

$$\therefore \int_{-\pi/2}^{\pi/2} f(x) dx = 0 \text{ (since } f(x) \text{ is an odd function)}$$

Example : If $f(x) = \begin{vmatrix} x^3 & \sin x \\ 1 & 2 \end{vmatrix}$, then $\int_{-a}^a f(x) dx$ is

- (a) 0 (b) $\frac{1}{2}$ (c) 3 (d) $-\frac{1}{2}$

$$\text{Sol. } \int_{-a}^a f(x) dx = \begin{vmatrix} \int_{-a}^a x^3 dx & \int_{-a}^a \sin x dx \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0$$

Hence (a) is correct answer.

MATRICES

1. INTRODUCTION TO MATRICES

A set of $(m \times n)$ numbers arranged in the form of an ordered set of m rows and n columns is called a matrix of order $m \times n$.

$$A = [a_{ij}]_{m \times n}$$

$$\text{or } A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \text{ is a matrix of order } m \times n.$$

NOTES:

The matrix is not a number. It has no numerical value. But it is an arrangement of numbers.



2. TYPES OF MATRICES

2.1 Row Matrix

A matrix having only one row is called a row matrix or a row vector.

e.g. $A = [1 \ 2 \ -1 \ -2]$ is a row matrix of order 1×4 .

2.2 Column Matrix

A matrix having only one column is called a column matrix or a column vector.

e.g. $A = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 2 \\ 5 \\ 4 \end{bmatrix}$ are column matrices or order 3×1 and 4×1 respectively.

2.3 Square Matrix

A matrix in which the number of rows is equal to the number of column, say $(n \times n)$ is called a square matrix of order n .

e.g. the matrix $\begin{bmatrix} 2 & 1 & -1 \\ 3 & -2 & 5 \\ 1 & 5 & -3 \end{bmatrix}$ is square matrix of order 3.

Sum of diagonal elements of a square matrix is called its trace ($\text{tr}(A)$). Here $\text{tr}(A) = 2 - 2 - 3 = -3$

2.4 Diagonal Matrix

A square matrix is called a diagonal matrix if all the elements, except those in the leading diagonal, are zero.

$A = [a_{ij}]_{n \times n}$, $a_{ij} = 0$ for all $i \neq j$

2.5 Scalar Matrix

A diagonal matrix in which all the diagonal elements are equal is called the scalar matrix.

A square matrix $A = [a_{ij}]_{n \times n}$ is called a scalar matrix if.

- (i) $a_{ij} = 0$ for all $i \neq j$ and
- (ii) $a_{ii} = C$ for all $i \in \{1, 2, \dots, n\}$

2.6 Identity or Unit Matrix

A square matrix each of whose diagonal element is unity and each of whose non diagonal element is equal to zero is called an identity or unit matrix.

A square matrix $A = [a_{ij}]_{n \times n}$ is called identity or unit matrix if

- (i) $a_{ij} = 0$ for all $i \neq j$, and
- (ii) $a_{ii} = 1$ for all $i \in \{1, 2, \dots, n\}$

The identity matrix of order n is denoted by I_n .

e.g. $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2.7 Null Matrix

A matrix whose all elements are zero is called a null matrix or a zero matrix, represented by O .

e.g. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

2.8 Upper Triangular Matrix

A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0 \ \forall \ i > j$.

e.g. $A = \begin{bmatrix} 5 & 4 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix}$

2.9 Lower Triangular Matrix

A square matrix $A = [a_{ij}]$ is called lower triangular if $a_{ij} = 0 \ \forall \ i < j$.

e.g. $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 4 & 5 & 3 \end{bmatrix}$

2.10 Singular Matrix

A square matrix with zero determinant is called a singular matrix.

3. EQUALITY OF MATRICES

Two matrices $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{r \times s}$ are equal if

- (i) $m = r$, i.e., the number of rows in A equals the number of rows in B .
- (ii) $n = s$, i.e., the number of columns in A equals the number of columns in B .



(iii) $a_{ij} = b_{ij}$ for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.

If two matrices A and B are equal, we write $A = B$, otherwise we write $A \neq B$.

4. SUM OF MATRICES

Let $A = [a_{ij}]$, $B = [b_{ij}]$ be matrices of the same order $m \times n$.

Then $C = A + B = [c_{ij}]$, is a matrix of order $m \times n$.

Where, $[c_{ij}] = [a_{ij} + b_{ij}]$

e.g. $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 & 2 \\ 5 & 1 & 9 \end{bmatrix}$

$$A + B = \begin{bmatrix} 1+7 & 2+3 & 4+2 \\ 0+5 & 5+1 & 3+9 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 6 \\ 5 & 6 & 12 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 1-7 & 2-3 & 4-2 \\ 0-5 & 5-1 & 3-9 \end{bmatrix} = \begin{bmatrix} -6 & -1 & 2 \\ -5 & 4 & -6 \end{bmatrix}$$

Properties of Matrix Addition

(i) **Matrix addition is commutative**

$$A + B = B + A$$

(ii) **Matrix addition is associative**

$$A + (B + C) = (A + B) + C.$$

5. SCALAR MULTIPLE OF A MATRIX

If A be a given matrix and k is any scalar number real or complex.

Then matrix kA is a matrix of same order, where all the elements of kA are k times of the corresponding elements of A.

e.g. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 2 & 4 \end{bmatrix}$

$$\text{Then } 3A = \begin{bmatrix} 3 \cdot 2 & 3 \cdot 3 & 3 \cdot 1 \\ 3 \cdot 5 & 3 \cdot 2 & 3 \cdot 4 \end{bmatrix} = \begin{bmatrix} 6 & 9 & 3 \\ 15 & 6 & 12 \end{bmatrix}$$

Properties of Multiplication by a Scalar

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrix of the same order and α and β are any scalars, then

(i) $\alpha(A + B) = \alpha A + \alpha B$

(ii) $(\alpha + \beta)A = \alpha A + \beta A$

(iii) $\alpha(\beta A) = (\alpha\beta)A$.

(iv) If A is a square matrix of order 'n'

$$\text{Then } |kA| = k^n |A|$$

6. MATRIX MULTIPLICATION

If $A = [a_{ij}]_{m \times p}$ and $B = [b_{jk}]_{p \times n}$

Then $A_{m \times p} \times B_{p \times n} = (AB)_{m \times n}$

or $C = AB = [c_{ik}]_{m \times n}$ where $c_{ik} = \sum_{j=1}^p a_{ij}b_{jk}$

For multiplication number of columns of first matrix should be equal to number of rows of second matrix.

i.e. $c_{ik} = a_{i1}b_{1k} + a_{i2}b_{2k} + \dots + a_{ip}b_{pk}$

In other words c_{ik} = Sum of the products of i^{th} row of A (having p elements) with k^{th} column of B (having p elements).

e.g. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}_{2 \times 3}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}_{3 \times 2}$

Compute AB and show that $AB \neq BA$. A is 2×3 type and B is 3×2 type and hence both AB and BA are defined because the number of columns in pre factor is equal to the number of rows in post factor.

Sol.

$$AB = \begin{bmatrix} 1 \cdot 2 - 2 \cdot 4 + 3 \cdot 2 & 1 \cdot 3 - 2 \cdot 5 + 3 \cdot 1 \\ -4 \cdot 2 + 2 \cdot 4 + 5 \cdot 2 & -4 \cdot 3 + 2 \cdot 5 + 5 \cdot 1 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}_{2 \times 2}$$

$$BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}_{2 \times 3} = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}_{3 \times 3}$$

Hence $AB \neq BA$.

7. PROPERTIES OF MATRIX MULTIPLICATION

(i) **Multiplication of matrices is distributive with respect to a addition of matrices.**

$$A(B + C) = AB + AC \text{ and } (A + B)C = AC + BC$$

(ii) **Matrix multiplication is associative if conformability is assured.**

$$\text{i.e. } A(BC) = (AB)C.$$

(iii) **The multiplication of matrices is not always commutative.**
i.e. AB is not always equal to BA.



- (iv) **Multiplication of a matrix A by a null matrix conformable with A for multiplication is a null matrix i.e. $AO = O$.**

In particular if A be a square matrix and O be square null matrix of the same order, then $OA = AO = O$.

- (v) **If $AB = O$ then it does not necessarily mean that $A = O$ or $B = O$.**

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

None of the matrices on the left is a null matrix whereas their products is a null matrix.

- (vi) **Multiplication of matrix A by a unit matrix I :**
Let A be a $m \times n$ matrix.

$$\text{Then } AI_n = A \quad \text{and } I_m A = A.$$

- (vii) **If A and B are square matrices of order 'n'**

$$\text{Then } |AB| = |A| |B|$$

- (viii) **Positive Integral Powers of Matrix**

Let A be any square matrix of order n.

$$\text{Then } A^2 = A.A$$

$$A^3 = A.A.A$$

$$A^m = A.A.A \dots m \text{ times}$$

All are square matrix of order n.

- (i) $A^m . A^n = (A.A.A \dots m \text{ times}) (A.A.A \dots n \text{ times})$
 $= A.A.A \dots (m+n) \text{ times}$
 $= A^{m+n}$

- (ii) $(A^m)^n = A^{mn}$

Also, we define $A^0 = I$

8. TRANSPOSE OF A MATRIX

If A be a given matrix of the order $m \times n$ then the matrix obtained by changing the rows of A into columns and columns of A into rows is called Transpose of matrix A and is denoted by A' or A^T . Hence the matrix A' is of order $n \times m$.

$$\text{e.g. } A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 5 & 9 \end{bmatrix}_{3 \times 2} \quad \text{then } A^T = A' = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 9 \end{bmatrix}_{2 \times 3}$$

Properties of Transpose

- (i) $(A')' = A$.
 (ii) $(kA)' = kA'$. k being a scalar.
 (iii) $(A+B)' = A' + B'$.
 (iv) $(AB)' = B'A'$.
 (v) $(ABC)' = C'B'A'$.

9. SYMMETRIC AND SKEW SYMMETRIC MATRICES

- (i) A square matrix $A = [a_{ij}]$ will be called **symmetric** if $A^T = A$.
 i.e. every ij^{th} element = ji^{th} element.

$$\text{e.g. } A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}_{3 \times 3}$$

- (ii) A square matrix $A = [a_{ij}]$ will be called **skew symmetric** if $A^T = -A$.

i.e. every ij^{th} element = $-(ji^{\text{th}} \text{ element})$.

- (iii) For any square matrix A, $A + A^T$ is symmetric and $A - A^T$ is skew - symmetric.

- (iv) Any square matrix can be uniquely expressed as a sum of a symmetric matrix and a skew-symmetric matrix.

$$\frac{1}{2}(A + A^T) \text{ is symmetric}$$

$$\text{and } \frac{1}{2}(A - A^T) \text{ is skew-symmetric}$$

- (v) Let A and B be symmetric matrices of the same order. Then the following hold:

- A^n is symmetric for all positive integers n.
- AB is symmetric if and only if $AB = BA$.
- $AB + BA$ is symmetric.
- $AB - BA$ is skew - symmetric

NOTES:

For a skew symmetric matrix :

$$a_{ii} = -a_{ii} \text{ for all values of } i$$

[i = j when elements are diagonals].

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$$2a_{ii} = 0 \quad \therefore \quad a_{ii} = 0$$

Hence the diagonal elements of skew symmetric matrix are zero.

e.g. $\begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{bmatrix}$ is a skew symmetric matrix

10. ADJOINT

If A is a square matrix, then transpose of a matrix made from cofactors of elements of A is called adjoint matrix of A. It's denoted by $\text{adj } A$.

Properties of Adjoint Matrix

- (i) $A \cdot (\text{Adj } A) = |A| I_n = (\text{adj } A) \cdot A$
- (ii) $|\text{adj } A| = |A|^{n-1}$
- (iii) $\text{adj}(\text{adj } A) = |A|^{n-2} A$
- (iv) $(\text{adj } A)^T = \text{adj}(A^T)$
- (v) $\text{adj}(AB) = (\text{adj } B) \cdot (\text{adj } A)$
- (iv) $\text{Adj}(A^{-1}) = (\text{adj } A)^{-1}$
- (vii) $|\text{adj}(\text{adj}(A))| = |A|^{(n-1)^2}$

11. INVERSE OF MATRIX (A)

11.1

A square matrix A of order n is said to be invertible or non-singular if there exists a square matrix B of order n such that

$$AB = I_n = BA$$

where I_n is the identity matrix of order n, B is called inverse of A and is denoted by A^{-1} .

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

11.2 Properties of Inverse Matrices

- (i) $(A^T)^{-1} = (A^{-1})^T$
- (ii) $(AB)^{-1} = B^{-1} A^{-1}$
- (iii) $(A^{-1})^{-1} = A$
- (iv) $|A^{-1}| = \frac{1}{|A|}$

$$(v) \quad (kA)^{-1} = \frac{1}{k} A^{-1} \text{ if } k \neq 0.$$

(vi) Let A, B, C be square matrix of the same order n. If A is non-singular matrix then

- (a) $AB = AC \Rightarrow B = C$ (left cancellation law)
- (b) $BA = CA \Rightarrow B = C$ (right cancellation law)

(vii) If A is non singular matrix such that A is symmetric then A^{-1} is also symmetric.

12. ELEMENTARY TRANSFORMATIONS

Any one of the following operations on a matrix is called an elementary transformation.

- (i) Interchanging any two rows (or column).
- (ii) Multiplication of the elements of any row (or column) by a non zero scalar quantity.
- (iii) Addition of constant multiple of the elements of any row (or column) to the corresponding element of any other row (or column).

Two matrices are said to be equivalent if one is obtained from the other by elementary transformation. The symbol \approx is used for equivalence.

Method to Find Inverse by Elementary Transformations :

Row Transformation

- (i) A^{-1} exists if $|A| \neq 0$.
- (ii) To find A^{-1} by row transformation, then we write $IA = A$.
- (iii) Apply row transformations to the pre-factor I on L.H.S. & to A on R.H.S. such that A becomes a unit matrix.
- (iv) Now equation becomes $BA = I$, so $B = A^{-1}$

13. SOLUTION OF A SYSTEM OF LINEAR EQUATION BY MATRIX METHOD

Consider a system of linear equation

$$a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n = b_2$$

$$\dots \dots \dots$$

$$\dots \dots \dots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots a_{nn}x_n = b_n$$



We can express these equations as a single matrix equation.

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix}$$

$$A \quad X \quad B$$

Let $|A| \neq 0$ so that A^{-1} exists uniquely

pre-multiplying both sides of $AX = B$ by A^{-1} we get

$$\begin{aligned} A^{-1}(AX) &= A^{-1}B & \Rightarrow & (AA^{-1})X = A^{-1}B \\ \Rightarrow IX &= A^{-1}B & \Rightarrow & X = A^{-1}B \end{aligned}$$

Criterion of Consistency

Let $AX = B$ be a system of n linear equation in n variables.

- If $|A| \neq 0$ then the system of the equations is consistent and has a unique solution given by $X = A^{-1}B$.
- If $|A| = 0$ and $(\text{adj } A)B = O$ then the system of equations is consistent and has infinitely many solutions except the case of parallel planes when there is no solution.
- If $|A| = 0$ and $(\text{adj } A)B \neq O$ then the system of equations is inconsistent i.e. it has no solution.

Homogeneous Equation

The system of equations $AX = B$ is said to be homogeneous if the constants $b_1, b_2, b_3, \dots, b_n$ are all zero. That is if the matrix B is a zero matrix and the system is of the form

$$AX = O$$

Where O is the null matrix of order $n \times 1$.

- If $|A| \neq 0$ then its only solution $X = 0$ is called trivial solution. ($x = y = z = 0$)
- If $|A| = 0$ then $AX = O$ have both trivial and non trivial type solutions. In this case number of solutions will be Infinite.
- The condition for
 $a_1x + b_1y + c_1z = 0$;
 $a_2x + b_2y + c_2z = 0$ and
 $a_3x + b_3y + c_3z = 0$
 to have non-zero or non-trivial solutions is :

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

14. SOME OTHER TYPES OF MATRICES

- (a) **Orthogonal Matrix** : A square matrix A is called an orthogonal matrix if the product of the matrix A and its transpose A' is identity matrix.

$$AA' = I$$

NOTES:

- If $AA' = I$ then $A^{-1} = A'$
- If A and B are orthogonal then AB is also orthogonal.
- All above properties are defined for square matrix only.
- Elements of all 3 rows (or columns) of orthogonal matrix of order 3×3 represent unit vectors.

- (b) **Idempotent Matrix** : A matrix ' A ' such that $A^2 = A$ is called idempotent..

- Only square matrix can be idempotent matrix.
- Identity matrix (unit matrix) is also idempotent matrix.

Example : $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Periodic Matrix** : A matrix ' A ' will be called a periodic matrix if $A^{k+1} = A$ where k is +ve integer and k is least positive integer for which $A^{k+1} = A$, then k is said to be the period of A .
- Nilpotent Matrix** : A matrix ' A ' will be called nilpotent matrix if $A^k = O$ (null matrix), k is least positive integer and k is called index of the nilpotent matrix.
- Involutory Matrix** : A matrix ' A ' will be called an involutory matrix if $A^2 = I$ (Unit Matrix). Unit matrix is also involutory matrix.



15. SOME IMPORTANT APPLICATIONS

15.1

Let $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_mx^m$ be a polynomial in x where $a_0, a_1, a_2, \dots, a_m$ are real numbers, such that $a_0 \neq 0$. If A is a non-singular matrix such that $f(A) = 0$, then

$$A^{-1} = \frac{-1}{a_0} (a_1 + a_2A + a_3A^2 + \dots + a_mA^{m-1})$$

15.2

To find the inverse of a square matrix A , or to express A^{-1} in terms of A , the concept of a characteristic polynomial of a square matrix and the much known Cayley-Hamilton Theorem are useful, especially for 2×2 and 3×3 matrices.

15.3

If A is a square matrix and I is the corresponding unit matrix, then the polynomial $|A - xI|$ in x is called characteristic polynomial of A and the equation $|A - xI| = 0$ is called the characteristic equation of the matrix A .

15.4

(Cayley-Hamilton) Every square matrix satisfies its characteristic equation; that is, if A is a square matrix of order n and

$$f(x) = |A - xI| = a_0 + a_1x + a_2x^2 + \dots + a_nx^n = 0$$

is its characteristic equation, then

$$f(A) = a_0I_n + a_1A + a_2A^2 + \dots + a_nA^n = O$$

Also if $a_0 \neq 0$, then

$$A^{-1} = \frac{-1}{a_0} (a_1I + a_2A + a_3A^2 + \dots + a_nA^{n-1})$$

Note that A^{-1} exists if and only if the constant term of the characteristic of A is non-zero.



SOLVED EXAMPLES

Example – 1

Find minors and cofactors of elements of the following determinants

$$(i) \begin{vmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & 3 & 0 \end{vmatrix} \quad (ii) \begin{vmatrix} -1 & 0 & 4 \\ -2 & 1 & 3 \\ 0 & -4 & 2 \end{vmatrix}$$

Sol. (i) $D = \begin{vmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 2 & 3 & 0 \end{vmatrix}$

Minors

$$M_{11} = \begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix};$$

$$= 0 - 3 = -3$$

$$M_{12} = \begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix};$$

$$= 0 - 2 = -2$$

$$M_{13} = \begin{vmatrix} 3 & 0 \\ 2 & 3 \end{vmatrix};$$

$$= 9 - 0 = 9$$

$$M_{21} = \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix};$$

$$= 0 - 6 = -6$$

$$M_{22} = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix};$$

$$= 0 - 4 = -4$$

$$M_{23} = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix};$$

$$= 0 - 2 = -2$$

Cofactors

$$A_{11} = (-1)^{1+1} M_{11}$$

$$= -3$$

$$A_{12} = (-1)^{1+2} M_{12}$$

$$= 2$$

$$A_{13} = (-1)^{1+3} M_{13}$$

$$= 9$$

$$A_{21} = (-1)^{2+1} M_{21}$$

$$= 6$$

$$A_{22} = (-1)^{2+2} M_{22}$$

$$= -4$$

$$A_{23} = (-1)^{2+3} M_{23}$$

$$= 2$$

$$M_{31} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix};$$

$$= 1 - 0 = 1$$

$$M_{32} = \begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix};$$

$$= 0 - 6 = -6$$

$$M_{33} = \begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix};$$

$$= 0 - 3 = -3$$

$$A_{31} = (-1)^{3+1} M_{31}$$

$$= 1$$

$$A_{32} = (-1)^{3+2} M_{32}$$

$$= 6$$

$$A_{33} = (-1)^{3+3} M_{33}$$

$$= -3$$

$$(ii) D = \begin{vmatrix} -1 & 0 & 4 \\ -2 & 1 & 3 \\ 0 & -4 & 2 \end{vmatrix}$$

Minors

$$M_{11} = \begin{vmatrix} 1 & 3 \\ -4 & 2 \end{vmatrix};$$

$$= 2 + 12 = 14$$

$$M_{12} = \begin{vmatrix} -2 & 3 \\ 0 & 2 \end{vmatrix};$$

$$= -4 - 0 = -4$$

$$M_{13} = \begin{vmatrix} -2 & 1 \\ 0 & -4 \end{vmatrix};$$

$$= 8 - 0 = 8$$

$$M_{21} = \begin{vmatrix} 0 & 4 \\ -4 & 2 \end{vmatrix};$$

$$= 0 + 16 = 16$$

$$M_{22} = \begin{vmatrix} -1 & 4 \\ 0 & 2 \end{vmatrix};$$

$$= -2 - 0 = -2$$

Cofactors

$$A_{11} = (-1)^{1+1} M_{11}$$

$$= 14$$

$$A_{12} = (-1)^{1+2} M_{12}$$

$$= 4$$

$$A_{13} = (-1)^{1+3} M_{13}$$

$$= 8$$

$$A_{21} = (-1)^{2+1} M_{21}$$

$$= -16$$

$$A_{22} = (-1)^{2+2} M_{22}$$

$$= -2$$



$$= -2 + 0 = -2 \quad = -2$$

$$M_{23} = \begin{vmatrix} -1 & 0 \\ 0 & -4 \end{vmatrix}; \quad A_{23} = (-1)^{2+3} M_{23}$$

$$= 4 - 0 = 4 \quad = -4$$

$$M_{31} = \begin{vmatrix} 0 & 4 \\ 1 & 3 \end{vmatrix}; \quad A_{31} = (-1)^{3+1} M_{31}$$

$$= 0 - 4 = -4 \quad = -4$$

$$M_{32} = \begin{vmatrix} -1 & 4 \\ -2 & 3 \end{vmatrix}; \quad A_{32} = (-1)^{3+2} M_{32}$$

$$= -3 + 8 = 5 \quad = -5$$

$$M_{33} = \begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix}; \quad A_{33} = (-1)^{3+3} M_{33}$$

$$= -1 + 0 = -1 \quad = -1$$

Example - 2

Show that $\begin{vmatrix} x-y & y-z & z-x \\ y-z & z-x & x-y \\ z-x & x-y & y-z \end{vmatrix} = 0$

Sol. $D = \begin{vmatrix} x-y & y-z & z-x \\ y-z & z-x & x-y \\ z-x & x-y & y-z \end{vmatrix}$

$C_3 \rightarrow C_3 + C_2 + C_1$ gives

$$D = \begin{vmatrix} x-y & y-z & 0 \\ y-z & z-x & 0 \\ z-x & x-y & 0 \end{vmatrix}$$

$$= 0$$

[\therefore all elements of C_3 are zero.]

Example - 3

Evaluate

$$(i) \begin{vmatrix} 16 & 29 & 35 \\ 50 & 100 & 110 \\ 82 & 158 & 180 \end{vmatrix}$$

$$(ii) \begin{vmatrix} 5 & 13 & 17 \\ 30 & 68 & 105 \\ 25 & 66 & 84 \end{vmatrix}$$

Sol. (i) $D = \begin{vmatrix} 16 & 29 & 35 \\ 50 & 100 & 110 \\ 82 & 158 & 180 \end{vmatrix}$

Using $\left(\frac{1}{10} R_2\right)$,

$$D = 10 \begin{vmatrix} 16 & 29 & 35 \\ 5 & 10 & 11 \\ 82 & 158 & 180 \end{vmatrix}$$

$R_1 = R_1 - 3R_2, R_3 \rightarrow R_3 - 16R_2$

$$D = 10 \begin{vmatrix} 1 & -1 & 2 \\ 5 & 10 & 11 \\ 2 & -2 & 4 \end{vmatrix} = 0$$

$\therefore (R_3 \equiv 2 \times R_1)$

$$(ii) D = \begin{vmatrix} 5 & 13 & 17 \\ 30 & 68 & 105 \\ 25 & 66 & 84 \end{vmatrix}$$

$R_2 \rightarrow R_2 - 6R_1, R_3 \rightarrow R_3 - 5R_1$

$$\therefore D = \begin{vmatrix} 5 & 13 & 17 \\ 0 & -10 & 3 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 5(10 - 3)$$

(Expanding along 1st column) = 35



Example – 4

Show that $\begin{vmatrix} 10 & 24 & 36 \\ 36 & 10 & 24 \\ 24 & 36 & 10 \end{vmatrix}$ is divisible by 35.

Sol. $D = \begin{vmatrix} 10 & 24 & 36 \\ 36 & 10 & 24 \\ 24 & 36 & 10 \end{vmatrix}$

$R_1 \rightarrow R_1 + R_2 + R_3$

$\therefore D = \begin{vmatrix} 70 & 70 & 70 \\ 36 & 10 & 24 \\ 24 & 36 & 10 \end{vmatrix}$

Using $\frac{1}{35} R_1$, we get

$D = 35 \begin{vmatrix} 2 & 2 & 2 \\ 36 & 10 & 24 \\ 24 & 36 & 10 \end{vmatrix}$

All elements in determinant are natural numbers

\therefore Its value will be whole number

\therefore D is multiple of 35 i.e. it is divisible by 35.

Example – 5

If x, y, z are different and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$,

then show that $1 + xyz = 0$

Sol. We have

$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$

$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$ (Using Property 6)

Take common x, y, z from R_1, R_2, R_3 resp.

$= (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

(Using $C_3 \leftrightarrow C_2$ and then $C_1 \leftrightarrow C_2$)

$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (1 + xyz)$

$= (1 + xyz) \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$

(Using $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$)

Taking out common factor $(y-x)$ from R_2 and $(z-x)$ from R_3 , we get

$\Delta = (1 + xyz)(y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix}$

$= (1 + xyz)(y-x)(z-x)(z-y)$ (on expanding along C_1)

Since $\Delta = 0$ and x, y, z are all different, i.e., $x-y \neq 0$,

$y-z \neq 0, z-x \neq 0$, we get $1 + xyz = 0$

Example – 6

If l, m, n are $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of an arithmetic progression

respectively, prove that $\begin{vmatrix} l & p & 1 \\ m & q & 1 \\ n & r & 1 \end{vmatrix} = 0$

Sol. In arithmetic progression

$T_1 = a$

$T_2 = a + d$

(where d is constant difference)

$T_3 = a + 2d$

$T_n = a + (n-1)d$

$\therefore l = T_p = a + (p-1)d = a + pd - d$



$$m = T_q = a + (q-1)d = a + qd - d$$

$$n = T_r = a + (r-1)d = a + rd - d$$

$$\therefore \text{L.H.S.} = \begin{vmatrix} l & p & 1 \\ m & q & 1 \\ n & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a+pd-d & p & 1 \\ a+qd-d & q & 1 \\ a+rd-d & r & 1 \end{vmatrix}$$

gives $C_1 \rightarrow C_1 - d \times C_2$

$$\text{L.H.S.} = \begin{vmatrix} a-d & p & 1 \\ a-d & q & 1 \\ a-d & r & 1 \end{vmatrix}$$

Using $\left(\frac{1}{a-d}\right) C_1$, we get

$$\text{L.H.S.} = (a-d) \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix}$$

$$= 0 \quad (\because C_1 \equiv C_3)$$

Example - 7

Without expanding the determinants, show that

$$(i) \begin{vmatrix} 1/x & x & yz \\ 1/y & y & zx \\ 1/z & z & xy \end{vmatrix} = 0 \quad (ii) \begin{vmatrix} 0 & b & c \\ -b & 0 & a \\ -c & -a & 0 \end{vmatrix} = 0$$

Sol. (i) $D = \begin{vmatrix} 1/x & x & yz \\ 1/y & y & zx \\ 1/z & z & xy \end{vmatrix}$

using xR_1, yR_2 and zR_3 , we get

$$D = \frac{1}{xyz} \begin{vmatrix} 1 & x^2 & xyz \\ 1 & y^2 & xyz \\ 1 & z^2 & xyz \end{vmatrix}$$

Using $\left(\frac{1}{xyz}\right) C_3$, we get

$$D = \frac{xyz}{xyz} \begin{vmatrix} 1 & x^2 & 1 \\ 1 & y^2 & 1 \\ 1 & z^2 & 1 \end{vmatrix}$$

$$= 0 \quad (\because C_1 \equiv C_3)$$

$$(ii) D = \begin{vmatrix} 0 & b & c \\ -b & 0 & a \\ -c & -a & 0 \end{vmatrix}$$

$$= (-1)^3 \begin{vmatrix} 0 & -b & -c \\ b & 0 & -a \\ c & a & 0 \end{vmatrix} \quad (\text{Taking } -1 \text{ common from } R_1, R_2 \& R_3)$$

$$\text{So } D = - \begin{vmatrix} 0 & -b & -c \\ b & 0 & -a \\ c & a & 0 \end{vmatrix}$$

$$= - \begin{vmatrix} 0 & b & c \\ -b & 0 & a \\ -c & -a & 0 \end{vmatrix} \quad (\text{Interchanging rows with columns \& vice-versa}).$$

$$= -D$$

$$\therefore 2D = 0 \quad \Rightarrow D = 0.$$

Example - 8

Show that

$$\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = (x-y)(y-z)(z-x)$$

Sol. $D = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

$R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$, gives



$$D = \begin{vmatrix} 0 & x-z & -y(x-z) \\ 0 & y-z & -x(y-z) \\ 1 & z & xy \end{vmatrix}$$

Using $\left(\frac{1}{z-x}\right)R_1$ and $\left(\frac{1}{y-z}\right)R_2$, we get

$$D = (z-x)(y-z) \begin{vmatrix} 0 & -1 & y \\ 0 & 1 & -x \\ 1 & z & xy \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_2$ gives

$$D = (z-x)(y-z) \begin{vmatrix} 0 & 0 & y-x \\ 0 & 1 & -x \\ 1 & z & xy \end{vmatrix}$$

Expanding along R_1

$$\begin{aligned} \therefore D &= (z-x)(y-z)(y-x) \begin{vmatrix} 0 & 1 \\ 1 & z \end{vmatrix} \\ &= (z-x)(y-z)(y-x)(0-1) \\ &= (x-y)(y-z)(z-x) \\ &= \text{R.H.S.} \end{aligned}$$

Example - 9

Show that

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$$= 2(a+b)(b+c)(c+a)$$

$$\text{Sol. } D = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$C_1 \rightarrow C_1 + C_3; C_2 \rightarrow C_2 + C_3$ gives

$$D = \begin{vmatrix} a+c & -(b+c) & -b \\ -(a+c) & b+c & -a \\ a+c & b+c & a+b+c \end{vmatrix}$$

Using $\left(\frac{1}{a+c}\right)C_1$ and $\left(\frac{1}{b+c}\right)C_2$ we get

$$D = (a+c)(b+c) \begin{vmatrix} 1 & -1 & -b \\ -1 & 1 & -a \\ 1 & 1 & a+b+c \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_2$ gives

$$D = (b+c)(c+a) \begin{vmatrix} 0 & 0 & -(a+b) \\ -1 & 1 & -a \\ 1 & 1 & a+b+c \end{vmatrix}$$

Expanding along R_1

$$\begin{aligned} &= (b+c)(c+a) \times [-(a+b)(-1-1)] \\ &= 2(a+b)(b+c)(c+a) \\ &= \text{R.H.S.} \end{aligned}$$

Example - 10

Show that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) = abc + bc + ca + ab$$

Sol. Taking out factors a, b, c common from R_1, R_2 and R_3 , we get

$$\text{L.H.S.} = abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have

$$\Delta = abc \begin{vmatrix} 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$



Taking $\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ common from R_1

$$= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b} + 1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c} + 1 \end{vmatrix}$$

Now applying $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$$

Expanding along C_3

$$= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) [1(1-0)]$$

$$= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = abc + bc + ca + ab = \text{R.H.S.}$$

Example – 11

Prove that

$$\Delta = \begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

Sol. Applying $R_1 \rightarrow R_1 - x R_2$ to Δ , we get

$$\Delta = \begin{vmatrix} a(1-x^2) & c(1-x)^2 & p(1-x^2) \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$$

Taking $1-x^2$ common from R_1

$$= (1-x^2) \begin{vmatrix} a & c & p \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - x R_1$, we get

$$\Delta = (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

= RHS

Example – 12

Find x , if

$$(i) \begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 4 & 6 & 9 \end{vmatrix} = 0 \quad (ii) \begin{vmatrix} 1 & 2x & 4x^2 \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Sol. (i) $\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 4 & 6 & 9 \end{vmatrix} = 0$

Expanding along R_1

$$\therefore 1(18-24) - x(9-16) + x^2(6-8) = 0$$

$$\therefore -6 + 7x - 2x^2 = 0$$

$$\therefore 2x^2 - 7x + 6 = 0$$

$$\therefore (2x-3)(x-2) = 0$$

$$\therefore 2x-3=0 \text{ or } x-2=0$$

$$\therefore x = \frac{3}{2} \text{ or } x = 2$$

(ii) Given $\begin{vmatrix} 1 & 2x & 4x^2 \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 0$

Method I: Here $a_{13} = (a_{12})^2$ (element in third column is equal to square of respective element in 2nd column)

We know, when two rows are identical then the determinant is zero.

$$\therefore \text{for } R_1 = R_2, 2x = 4 \text{ i.e. } x = 2$$

$$\therefore \text{for } R_1 = R_3, 2x = 1 \text{ i.e. } x = \frac{1}{2}$$

$$\therefore \text{solution is } x = \frac{1}{2}, 2$$



Method II: if we expand the given determinant by R_1 , we get

$$1(4-16)-2x(1-16)+4x^2(1-4)=0$$

$$\therefore -12-2x+32x-12x^2=0$$

$$\therefore -12x^2+30x-12=0$$

$$\therefore 4x^2-10x+4=0$$

$$\therefore (4x-2)(x-2)=0$$

$$\therefore 4x-2=0 \text{ or } x-2=0$$

$$\therefore x=\frac{1}{2} \text{ or } x=2$$

Example – 13

Solve the following equations using Cramer's Rule

$$x+2y-z=3, 3x-y+2z=1,$$

$$2x-2y+3z=2$$

Sol. The given equations are

$$x+2y-z=3$$

$$3x-y+2z=1$$

$$2x-2y+3z=2$$

$$D = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \end{vmatrix}$$

Expanding along R_1

$$= 1(-3+4) - 2(9-4) - 1(-6+2)$$

$$= 1 - 10 + 4 = -5$$

$$D_x = \begin{vmatrix} 3 & 2 & -1 \\ 1 & -1 & 2 \\ 2 & -2 & 3 \end{vmatrix}$$

Expanding along R_1

$$= 3(-3+4) - 2(3-4) - 1(-2+2)$$

$$= 5$$

$$D_y = \begin{vmatrix} 1 & 3 & -1 \\ 3 & +1 & 2 \\ 2 & 2 & 3 \end{vmatrix}$$

Expanding along R_1

$$= 1(3-4) - 3(9-4) - 1(6-2)$$

$$= -20$$

$$D_z = \begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & 1 \\ 2 & -2 & 2 \end{vmatrix}$$

$$= 1(-2+2) - 2(6-2) + 3(-6+2)$$

$$= -20$$

By Cramer's Rule

$$x = \frac{D_x}{D} = \frac{5}{-5} = -1$$

$$y = \frac{D_y}{D} = \frac{-20}{-5} = 4$$

$$z = \frac{D_z}{D} = \frac{-20}{-5} = 4$$

$$\therefore \text{Solution is } x=-1, y=4, z=4$$

Example – 14

Show that the following equations are consistent

$$2x+3y+1=0, x+2y+1=0, x+y=0$$

Sol. Given

$$2x+3y+1=0$$

$$x+2y+1=0$$

$$x+y=0$$

By condition of consistency

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\text{Now, } \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$ gives

$$= \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0 \quad (\because R_1 \equiv R_3)$$

\therefore The given equations are consistent.



Example – 15

Find k, if the following equations are consistent

$$(k-2)x + (k-1)y = 17,$$

$$(k-1)x + (k-2)y = 18,$$

$$x + y = 5$$

Sol. The given equations are

$$(k-2)x + (k-1)y - 17 = 0$$

$$(k-1)x + (k-2)y - 18 = 0$$

$$x + y - 5 = 0$$

∴ The equations are consistent

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} k-2 & k-1 & -17 \\ k-1 & k-2 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 - R_2$ gives

$$= \begin{vmatrix} -1 & 1 & 1 \\ k-1 & k-2 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$$

$C_2 \rightarrow C_2 + C_1, C_3 \rightarrow C_3 + C_1$ gives

$$\begin{vmatrix} -1 & 0 & 0 \\ k-1 & 2k-3 & k-19 \\ 1 & 2 & -4 \end{vmatrix}$$

Expanding along R_1

$$\therefore -1(-8k + 12 - 2k + 38) = 0$$

$$\therefore 10k - 50 = 0$$

$$\therefore k = 5$$

Example – 16

Find k if the area of the triangle ABC is 35 sq. units, where $A = (2, 6)$, $B = (5, 4)$ and $C = (k, 4)$

Sol. Area = $\frac{1}{2} \begin{vmatrix} 2 & 6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = 35$ sq. units

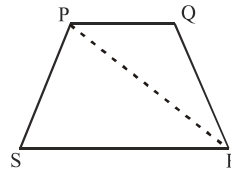
$$\Rightarrow \frac{1}{2} (k-5) 2 = \pm 35$$

$$\Rightarrow k-5 = \pm 35$$

$$\Rightarrow k = 40, -30.$$

Example – 17

Find the area of the quadrilateral whose vertices are $P(-3, 1)$, $Q(1, -1)$, $R(2, 1)$, $S(0, 3)$.



Sol.

$$A(PQRS) = A(\Delta PQR) + A(\Delta PRS)$$

$$A(\Delta PQR) = \frac{1}{2} \begin{vmatrix} -3 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [-3(-1-1) - 1(1-2) + 1(1+2)]$$

$$= \frac{1}{2} (6 + 1 + 3)$$

$$= 5 \text{ Sq. units}$$

$$A(\Delta PRS) = \frac{1}{2} \begin{vmatrix} -3 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [-3(1-3) - 1(2-0) + 1(6-0)]$$

$$= \frac{1}{2} [6 - 2 + 6]$$

$$= 5 \text{ sq. units}$$

$$\therefore A(PQRS) = 5 + 5 = 10 \text{ Sq. units.}$$



Example – 18

The sum of first and second numbers is greater than the third number by 5. The sum of first and third numbers is more than the second number by 7. The sum of second and third numbers is greater than the first number by 2. Find such three numbers.

Sol. Let the numbers be x, y, z respectively. We get the following equations.

$$x + y - z = 5$$

$$x - y + z = 7$$

$$-x + y + z = 2$$

$$\therefore D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 2(-1-1)$$

$$= -4$$

$$D_x = \begin{vmatrix} 5 & 1 & -1 \\ 7 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 12 & 0 & 0 \\ 7 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 12(-1-1)$$

$$= -24$$

$$D_y = \begin{vmatrix} 1 & 5 & -1 \\ 1 & 7 & 1 \\ -1 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(7-2) - 5(1+1) - 1(2+7)$$

$$= -14$$

$$D_z = \begin{vmatrix} 1 & 1 & 5 \\ 1 & -1 & 7 \\ -1 & 1 & 2 \end{vmatrix}$$

Expanding along R_1

$$= 1(-2-7) - 1(+2+7) + 5(1-1)$$

$$= -18$$

By Cramer's Rule

$$x = \frac{D_x}{D} = \frac{-24}{-4} = 6$$

$$y = \frac{D_y}{D} = \frac{-14}{-4} = \frac{7}{2}$$

$$z = \frac{D_z}{D} = \frac{-18}{-4} = \frac{9}{2}$$

\therefore The numbers are $6, \frac{7}{2}$ and $\frac{9}{2}$ respectively.

Example – 19

If $A = \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$, find matrix C such that

$A + B + C = 0$, where 0 is the zero matrix.

Sol. Given, $A + B + C = 0$

$$\therefore C = -[A + B]$$

$$A + B = \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$$

$$C = -[A + B] = -\begin{bmatrix} 8 & 4 \\ -2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} -8 & -4 \\ 2 & -4 \\ -4 & -4 \end{bmatrix}$$

Example – 20

Find matrices A and B , where

$$2A - B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \text{ and } A + 3B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Sol. Given $2A - B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$... (i)

$$A + 3B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \dots \text{(ii)}$$

From $3 \times \text{(i)} + \text{(ii)}$, we get



$$7A = \begin{bmatrix} 3 & -3 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & -3+1 \\ 0-1 & 3+0 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\therefore A = \frac{1}{7} \begin{bmatrix} 3 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 3/7 & -2/7 \\ -1/7 & 3/7 \end{bmatrix}$$

$$\therefore B = 2A - \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad \dots (i)$$

$$\therefore B = 2 \begin{bmatrix} 3/7 & -2/7 \\ -1/7 & 3/7 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6/7 & -4/7 \\ -2/7 & 6/7 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/7 & 3/7 \\ -2/7 & -1/7 \end{bmatrix}$$

Example - 21

If $A_\alpha = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, show that $A_\alpha \cdot A_\beta = A_{\alpha+\beta}$

Sol. $A_\alpha = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

then $A_\beta = \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$

$$A_\alpha \cdot A_\beta = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & \cos \alpha \sin \beta + \sin \alpha \cos \beta \\ -\sin \alpha \cos \beta - \cos \alpha \sin \beta & -\sin \alpha \sin \beta + \cos \alpha \cos \beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha + \beta) & \sin(\alpha + \beta) \\ -\sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix}$$

$$= A_{(\alpha+\beta)}$$

Example - 22

If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$, verify that $|AB| = |A| |B|$.

Sol. $AB = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 2+3 & 4-2 \\ 0+9 & 0-6 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 5 & 2 \\ 9 & -6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 6 - 0 = 6$$

$$|B| = \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -2 - 6 = -8$$

$$|AB| = \begin{vmatrix} 5 & 2 \\ 9 & -6 \end{vmatrix} = -30 - 18 = -48$$

$$\text{Also, } |A| \cdot |B| = 6(-8) = -48$$

Hence, $|AB| = |A| \cdot |B|$ is verified.

Example - 23

If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$

Sol. $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$$\therefore A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5 \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25+0 \\ -20+20-0 & 24-10-14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore A^2 - 4A + 3I = 0$$

Example – 24

If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

show that $(A+B)(A-B) \neq A^2 - B^2$.

Sol. $(A+B)(A-B) = A^2 - AB + BA - B^2$

$$AB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

And, $BA = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore AB \neq BA$$

$$\therefore -AB + BA \neq 0$$

$$\therefore (A+B)(A-B) \neq A^2 - B^2$$

Example – 25

If $A = \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & x \\ y & 0 \end{bmatrix}$, and $(A+B)(A-B) = A^2 - B^2$,

find x and y

Sol. Condition given

$$(A+B)(A-B) = A^2 - B^2$$

$$\therefore A^2 - AB + BA - B^2 = A^2 - B^2$$

$$\therefore -AB + BA = 0$$

$$\therefore AB = BA$$

$$\therefore \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} 1 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 1 & x \\ y & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -3+2y & -3x+0 \\ 2-4y & 2x+0 \end{bmatrix} = \begin{bmatrix} -3+2x & 2-4x \\ -3y+0 & 2y+0 \end{bmatrix}$$

Comparing corresponding elements.

$$-3x = 2 - 4x \text{ and } -3y = 2 - 4y$$

$$\therefore x = 2 \quad y = 2$$

$$[\therefore x = y = 2]$$

Example – 26

If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, find A^3

Sol. $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\therefore A^3 = A \cdot A^2 = A \cdot I = A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

Example – 27

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$,

show that AB and BA are both singular matrices.

Sol. $AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1-6-6 & -1+4+3 & 1-2+0 \\ 2-12-12 & -2+8+6 & 2-4+0 \\ 1-6-6 & -1+4+3 & 1-2+0 \end{bmatrix}$$



$$= \begin{bmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{vmatrix} = 0 \quad (\because R_1 = R_3)$$

Similarly,

$$BA = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+1 & 2-4+2 & 3-6+3 \\ -3+4-1 & -6+8-2 & -9+12-3 \\ -2+2+0 & -4+4+0 & -6+6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore |BA| = 0 \quad (\because \text{it is zero matrix})$$

Hence, AB and BA both are singular.

Example – 28

Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a

symmetric and a skew symmetric matrix.

Sol. Here

$$B' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(B+B') = \frac{1}{2} \begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix}$$

$$\text{Now } P' = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} = P$$

Thus $P = \frac{1}{2}(B+B')$ is a symmetric matrix.

Also, let

$$Q = \frac{1}{2}(B-B') = \frac{1}{2} \begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$

$$\text{Then } Q' = \begin{bmatrix} 0 & \frac{1}{2} & \frac{5}{2} \\ \frac{-1}{2} & 0 & -3 \\ \frac{-5}{2} & 3 & 0 \end{bmatrix} = -Q$$

Thus $Q = \frac{1}{2}(B-B')$ is a skew symmetric matrix.

Now

$$P+Q = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = B$$

Thus, B is represented as the sum of a symmetric and a skew symmetric matrix.



Example – 29

The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Sol. Let first, second and third numbers be denoted by x , y and z , respectively. Then, according to given conditions, we have

$$x + y + z = 6$$

$$y + 3z = 11$$

$$x + z = 2y \text{ or } x - 2y + z = 0$$

This system can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

Here $|A| = 1(1+6) - (0-3) + (0-1) = 9 \neq 0$. Now we find $\text{adj } A$

$$A_{11} = 1(1+6) = 7, A_{12} = -(0-3) = 3, A_{13} = -1$$

$$A_{21} = -(1+2) = -3, A_{22} = 0, A_{23} = -(-2-1) = 3$$

$$A_{31} = (3-1) = 2, A_{32} = -(3-0) = -3, A_{33} = (1-0) = 1$$

$$\text{Hence } \text{adj } A = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\text{Thus } A^{-1} = \frac{1}{|A|} \text{adj}(A) = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

Since $X = A^{-1}B$

$$X = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 42-33+0 \\ 18+0+0 \\ -6+33+0 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Thus $x = 1, y = 2, z = 3$

Example – 30

$$\text{Use product } \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \text{ to solve the}$$

system of equations

$$x - y + 2z = 1$$

$$2y - 3z = 1$$

$$3x - 2y + 4z = 2$$

$$\text{Sol. Consider the product } \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Hence } \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Now, given system of equations can be written, in matrix form, as follows

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

Hence $x = 0, y = 5$ and $z = 3$



Example – 31

Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$.

Find P such that $BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

Sol. Given $BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

Pre-multiplying both sides by B^{-1}

$$B^{-1} BPA = B^{-1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow IPA = B^{-1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow PA = B^{-1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \dots (i)$$

To find B^{-1} .

Now $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

$$|B| = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 8 - 9 = -1 \neq 0. \text{ As } |B| \neq 0$$

so it is non-singular matrix and hence inverse of B exists.

$$\Rightarrow B^{-1} = \frac{\text{Adj.} B}{|B|} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$$

NOTES :

For a 2×2 matrix, adjoint can be obtained by swapping diagonal elements and changing the sign of non-diagonal elements.

Now from (i),

$$PA = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow PA = \begin{bmatrix} -4 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix}$$

Post-multiplying both sides by A^{-1}

$$PAA^{-1} = \begin{bmatrix} -4 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix} A^{-1}$$

$$\Rightarrow PI = \begin{bmatrix} -4 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix} A^{-1}$$

$$\therefore P = \begin{bmatrix} -4 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix} A^{-1} \dots (ii)$$

For A^{-1} :

since $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$. Now $|A| = -1 \neq 0$

\Rightarrow it is non-singular matrix and hence A^{-1} exists

$$\text{adj.}(A) = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 1 \\ -2 & -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj.} A}{|A|} = \begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$$

Now From (ii),

$$P = \begin{bmatrix} -4 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix} \times \begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$$

$$\Rightarrow P = \begin{bmatrix} -4 & 7 & -7 \\ 3 & -5 & 5 \end{bmatrix}$$



Example – 32

Solve the system of equations :

$$x + 2y + z = 2$$

$$2x - 3y + 4z = 1$$

$$3x + 6y + 3z = 6$$

Sol. 1st and 3rd equations are integral multiple of each other.
(dependent equations)

$$\Rightarrow D = D_1 = D_2 = D_3 = 0$$

\Rightarrow infinite solutions

consider $x + 2y + z = 2$

$$2x - 3y + 4z = 1$$

Let $z = k$

$$\Rightarrow \begin{cases} x + 2y = 2 - k \\ 2x - 3y = 1 - 4k \end{cases}$$

$$\Rightarrow y = \frac{3+2k}{7} \text{ and } x = \frac{8-11k}{7}$$

$$\text{Hence : } \left[x = \frac{8-11k}{7}, y = \frac{3+2k}{7} \text{ and } z = k \right]$$

where k is an arbitrary constant.

Example – 33

Obtain the inverse of the following matrix using elementary operations

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Sol. Write $A = IA$, i.e.,

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\text{or } \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \text{ (applying } R_1 \leftrightarrow R_2)$$

$$\text{or } \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \text{ (applying } R_3 \rightarrow R_3 - 3R_1)$$

$$\text{or } \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \text{ (applying } R_1 \rightarrow R_1 - 2R_2)$$

$$\text{or } \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A \text{ (applying } R_3 \rightarrow R_3 + 5R_2)$$

$$\text{or } \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \text{ (applying } R_3 \rightarrow \frac{1}{2} R_3)$$

$$\text{or } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \text{ (applying } R_1 \rightarrow R_1 + R_3)$$

$$\text{or } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \text{ (applying } R_2 \rightarrow R_2 - 2R_3)$$

$$\text{Hence } A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$$



Example – 34

If M is a 3×3 matrix, where $M^T M = I$ and $\det(M) = 1$, then prove that $\det(M - I) = 0$.

Sol. $(M - I)^T = M^T - I = M^T - M^T M = M^T (I - M)$

$$\Rightarrow |M - I|^T = |M - I| = |M^T| |I - M|$$

$$= |I - M| \Rightarrow |M - I| = 0.$$

Alternate Method :

$$\det(M - I) = \det(M - I) \det(M^T)$$

$$= \det(MM^T - M^T)$$

$$= \det(I - M^T) = -\det(M^T - I)$$

$$= -\det(M - I)^T = -\det(M - I)$$

$$\Rightarrow \det(M - I) = 0.$$

Example – 35

If S is a skew-symmetric matrix of order n and $I + S$ is non-singular, then prove that

$A = (I - S)(I + S)^{-1}$ is an orthogonal matrix of order n .

Sol. $A^T = [(I + S)^T]^{-1} [I - S]^T$

$$= (I - S)^{-1} (I + S),$$

[since $S^T = -S$; S being skew symmetric].

$$\therefore A^T A = (I - S)^{-1} (I + S) (I - S) (I + S)^{-1}$$

$$= (I - S)^{-1} (I - S) (I + S) (I + S)^{-1},$$

$$\text{since } (I + S)(I - S) = (I - S)(I + S)$$

$$= I$$

$\therefore A$ is orthogonal, $I - S$ is a square matrix of order n .

$\therefore A = (I - S)(I + S)^{-1}$ is a square matrix of order n .

Example – 36

If A and B are n -rowed non-zero square matrix such that $AB = 0$, then show that both A and B are singular. If both A and B are singular and $AB = 0$, does it follow that $BA = 0$.

Justify your answer.

Sol. (i) $AB = 0$ and A non-singular implies

$$A^{-1}AB = A^{-1}(0) = 0$$

$$\Rightarrow (A^{-1}A)B = 0$$

$$\Rightarrow IB = 0 \Rightarrow B = 0 \text{ [Note true]}$$

(ii) $AB = 0$ and B non-singular implies

$$ABB^{-1} = 0(B)^{-1} = 0 \Rightarrow AI = 0$$

$$\Rightarrow A = 0 \text{ [Not true]}$$

\therefore Both A and B are singular.

(iii) Consider the counter example

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\text{whereas } BA = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq 0$$

Example – 37

Let A and B be matrix of order n . Prove that if $(I - AB)$ is invertible, then $(I - BA)$ is also invertible and $(I - BA)^{-1} = I + B(I - AB)^{-1}A$.

Sol. $I - BA = BIB^{-1} - BAB^{-1}$

$$= B(I - AB)B^{-1} \quad \dots(i)$$

$$\text{Hence, } |I - BA| = |B| |I - AB| |B^{-1}|$$

$$= |I - AB| |B| |B^{-1}|$$

$$= |I - AB| |B| |B^{-1}|$$

$$= |I - AB| \quad \dots(ii)$$

$$\text{Since } |B| |B^{-1}| = |BB^{-1}| = |I| = 1$$

If $I - AB$ is invertible, $|I - AB|$ has to be non-zero.

Hence, $|I - BA| \neq 0$ and therefore $I - BA$ is also invertible

$$\text{Now } (I - BA) \{I + B(I - AB)^{-1}A\}$$

$$= (I - BA) + (I - BA)B(I - AB)^{-1}A$$

$$= (I - BA) + \{B(I - AB)B^{-1}\}B(I - AB)^{-1}A$$

(Using (i))

$$= (I - BA) + B(I - AB)(I - AB)^{-1}A$$

$$= I - BA + BA = I$$

$$\text{Hence, } (I - BA)^{-1} = I + B(I - AB)^{-1}A$$



Example – 38

If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, prove that

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix} \text{ for all } n \in \mathbb{N}$$

Sol. Consider $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

$$\text{for } n = 1, A^1 = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix},$$

it is true as given

$\therefore A^n$ is true for $n = 1$

Let A^n is true for $n = r$, where $r \in \mathbb{N}$

$$\therefore A^r = \begin{bmatrix} \cos r\theta & \sin r\theta \\ -\sin r\theta & \cos r\theta \end{bmatrix}$$

then,

$$A^{r+1} = A \cdot A^r$$

$$= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos r\theta & \sin r\theta \\ -\sin r\theta & \cos r\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta \cos r\theta - \sin \theta \sin r\theta & \cos \theta \sin r\theta + \sin \theta \cos r\theta \\ -\sin \theta \cos r\theta - \cos \theta \sin r\theta & -\sin \theta \sin r\theta + \cos \theta \cos r\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta + r\theta) & \sin(\theta + r\theta) \\ -\sin(\theta + r\theta) & \cos(\theta + r\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(r+1)\theta & \sin(r+1)\theta \\ -\sin(r+1)\theta & \cos(r+1)\theta \end{bmatrix}$$

i.e. A^n is true for $n = r + 1$,

$\therefore A^n$ is true for $n = 1$

And A^n is true for $n = r + 1$, if it is true for $n = r$

$\therefore A^n$ is true for all $n \in \mathbb{N}$

$$\therefore A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix} \text{ for all } n \in \mathbb{N}.$$

Example – 39

$$A = \begin{bmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}, ab \neq 1$$

If there is vector matrix X , such that $AX = U$ has infinitely many solutions, then prove that $BX = V$ cannot have unique solution. If $afd \neq 0$ then prove that $BX = V$ has no solution.

Sol. $AX = U$ has infinite solutions

$$\Rightarrow |A| = 0$$

$$\begin{vmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{vmatrix} = 0$$

$$\Rightarrow ab = 1 \text{ or } c = d$$

$$\text{and } |A_1| = \begin{vmatrix} a & 0 & f \\ 1 & c & g \\ 1 & d & h \end{vmatrix} = 0$$

$\Rightarrow g = h$; [Here A_1 is actually D_1 for A : Cramer's Rule in Determinants section]

$$|A_2| = \begin{vmatrix} a & f & 1 \\ 1 & g & b \\ 1 & h & b \end{vmatrix} = 0 \Rightarrow g = h$$

$$\Rightarrow |A_3| = \begin{vmatrix} f & 0 & 1 \\ g & c & b \\ h & d & b \end{vmatrix} = 0 \Rightarrow g = h, c = d$$

$$\Rightarrow c = d \text{ and } g = h$$

So, for infinite solutions $c = d$ and $g = h$

$$BX = V$$

$$|B| = \begin{vmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{vmatrix} = 0 \text{ (Since } C_2 \text{ and } C_3 \text{ are equal)}$$

$$\Rightarrow BX = V \text{ has no unique solution}$$

$$\text{and } |B_1| = \begin{vmatrix} a^2 & 1 & 1 \\ 0 & d & c \\ 0 & g & h \end{vmatrix} = 0 \text{ (since } c = d, g = h)$$



$$|B_2| = \begin{vmatrix} a & a^2 & 1 \\ 0 & 0 & c \\ f & 0 & h \end{vmatrix} = a^2cf = a^2df \quad (\text{since } c = d)$$

$$|B_3| = \begin{vmatrix} a & 1 & a^2 \\ 0 & d & 0 \\ f & g & 0 \end{vmatrix} = a^2df$$

Since if $adf \neq 0$ then $|B_2| = |B_3| \neq 0$. Hence no solution exists.

Example – 40

Prove that the inverse of $\begin{pmatrix} A & 0 \\ B & C \end{pmatrix}$ is $\begin{pmatrix} A^{-1} & 0 \\ -C^{-1}BA^{-1} & C^{-1} \end{pmatrix}$

where A, C are non-singular matrix and hence find the

$$\text{inverse of } \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Sol. First part :

$$\text{As } \begin{pmatrix} A & 0 \\ B & C \end{pmatrix} \begin{pmatrix} A^{-1} & 0 \\ -C^{-1}BA^{-1} & C^{-1} \end{pmatrix}$$

$$\begin{pmatrix} AA^{-1} & 0 \\ BA^{-1} - CC^{-1}BA^{-1} & CC^{-1} \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

$$\text{and } \begin{pmatrix} A^{-1} & 0 \\ -C^{-1}BA^{-1} & C^{-1} \end{pmatrix} \begin{pmatrix} A & 0 \\ B & C \end{pmatrix}$$

$$\begin{pmatrix} A^{-1}A & 0 \\ -C^{-1}B + C^{-1}B & C^{-1}C \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

Hence $\begin{pmatrix} A^{-1} & 0 \\ C^{-1}BA^{-1} & C^{-1} \end{pmatrix}$ is the inverse of $\begin{pmatrix} A & 0 \\ B & C \end{pmatrix} = I$.

Second Part :

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} A & 0 \\ B & C \end{pmatrix}$$

$$\text{where } A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

$$\text{Inverse of } \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$$\text{since } C^{-1}BA^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$



EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Properties of determinants

1. If $P = \begin{vmatrix} 2 & 1 & 0 \\ 3 & 1 & 2 \\ 5 & 2 & 3 \end{vmatrix}$, then $\begin{vmatrix} 2 & 2 & 0 \\ 9 & 6 & 6 \\ 5 & 4 & 3 \end{vmatrix}$ is equal to

- (a) $2P$ (b) $3P$
(c) $5P$ (d) $6P$

2. $\begin{vmatrix} 11 & 12 & 13 \\ 12 & 13 & 14 \\ 13 & 14 & 15 \end{vmatrix}$ is equal to

- (a) 1 (b) 0
(c) -1 (d) 67

3. If every element of a third order determinant of value Δ is multiplied by 5, then the value of new determinant is

- (a) Δ (b) 5Δ
(c) 25Δ (d) 125Δ

4. $\Delta = \begin{vmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{vmatrix}$ is equal to

- (a) 1 (b) -1
(c) zero (d) 2

5. The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$, ω being a cube root of unity, is

- (a) 0 (b) 1
(c) ω^2 (d) ω

6. $\Delta = \begin{vmatrix} 0 & p-q & a-b \\ q-p & 0 & x-y \\ b-a & y-x & 0 \end{vmatrix}$ is equal to

- (a) 0 (b) $a+b$
(c) $x+y$ (d) $p+q$

7. If α, β & γ are the roots of the equation $x^3 + px + q = 0$ then the value of the determinant

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} =$$

- (a) p (b) q
(c) $p^2 - 2q$ (d) none

8. Given a, b, c are in A.P. Then determinant

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$
 in its simplified form is :

- (a) $x^3 + 3ax + 7c$ (b) 0
(c) 15 (d) $10x^2 + 5x + 2c$

9. If $a + b + c = 0$, one root of :

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$
 is

- (a) $x = 1$ (b) $x = 2$
(c) $x = a^2 + b^2 + c^2$ (d) $x = 0$

10. The determinant $\begin{vmatrix} b_1 + c_1 & c_1 + a_1 & a_1 + b_1 \\ b_2 + c_2 & c_2 + a_2 & a_2 + b_2 \\ b_3 + c_3 & c_3 + a_3 & a_3 + b_3 \end{vmatrix} =$

(a) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (b) $2 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

(c) $3 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (d) none of these



11. If
$$\begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \end{vmatrix},$$
 then the

value of $\sum_{r=1}^{n-1} \Delta r$:

- (a) depends only on a
- (b) depends only on n
- (c) depends both on a and n
- (d) is independent of both a and n.

Algebra of matrices

12. A matrix $A = [a_{ij}]$ of order 2×3 whose elements are such that $a_{ij} = i + j$ is -

- (a) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$
- (b) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 4 & 3 \end{bmatrix}$
- (c) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 5 & 3 \end{bmatrix}$
- (d) none of these

13. If A and B are 3×3 matrices, then $AB = O$ implies :

- (a) $A = O$ and $B = O$
- (b) $|A| = 0$ and $|B| = 0$
- (c) either $|A| = 0$ or $|B| = 0$
- (d) $A = O$ or $B = O$

14. If X and Y two matrices are such that

$X - Y = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $X + Y = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ then Y is given by

- (a) $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$
- (b) $\begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix}$
- (c) $\begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix}$
- (d) None of these

15. If $A + B = \begin{bmatrix} 7 & 4 \\ 8 & 9 \end{bmatrix}$ and $A - B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ then the value of A is-

- (a) $\begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix}$
- (b) $\begin{bmatrix} 4 & 3 \\ 4 & 6 \end{bmatrix}$
- (c) $\begin{bmatrix} 6 & 2 \\ 8 & 6 \end{bmatrix}$
- (d) $\begin{bmatrix} 7 & 6 \\ 8 & 6 \end{bmatrix}$

16. If $A = [a_{ij}]$ is a square matrix of order $n \times n$ and k is a scalar, then $|kA| =$

- (a) $k^n |A|$
- (b) $k |A|$
- (c) $kn^{-1} |A|$
- (d) none of these

17. If A, B, C are matrices of order 1×3 , 3×3 and 3×1 respectively, the order of ABC will be -

- (a) 3×3
- (b) 1×3
- (c) 1×1
- (d) 3×1

18. The order of $[xyz] \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is

- (a) 3×1
- (b) 1×1
- (c) 1×3
- (d) 3×3

19. If $[1 \times 2] \begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 2 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -1 \end{bmatrix} = O$, then the value of x is

- (a) -1
- (b) 0
- (c) 1
- (d) 2

20. If $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ then

- (a) $x = 2, y = 1$
- (b) $x = 1, y = 2$
- (c) $x = 3, y = 2$
- (d) $x = 2, y = 3$

21. If $A = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$ then element a_{21} of A^2 is -

- (a) 22
- (b) -15
- (c) -10
- (d) 7



22. If $E(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then value of $E(\alpha) \cdot E(\beta)$ is -
- (a) $E(0^\circ)$ (b) $E(90^\circ)$
(c) $E(\alpha + \beta)$ (d) $E(\alpha - \beta)$
23. The root of the equation $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ is
- (a) $1/3$ (b) $-1/3$
(c) 0 (d) 1
24. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, the $A^4 =$
- (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
25. If $A = \begin{bmatrix} p & q \\ -q & p \end{bmatrix}$, $B = \begin{bmatrix} r & s \\ -s & r \end{bmatrix}$ then
- (a) $AB = BA$ (b) $AB \neq BA$
(c) $AB = -BA$ (d) none of these
26. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and a and b are arbitrary constants then $(aI + bA)^2 =$
- (a) $a^2I + abA$ (b) $a^2I + 2abA$
(c) $a^2I + b^2A$ (d) none of these
27. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ then $(AB)^T$ equals -
- (a) $\begin{bmatrix} 5 & 16 \\ 9 & 16 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 9 \\ 16 & 12 \end{bmatrix}$
(c) $\begin{bmatrix} 5 & 9 \\ 4 & 3 \end{bmatrix}$ (d) none of these
28. If $A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ then $B^T A^T$ is equal to -
- (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
29. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ then which of the following statements is true -
- (a) $AB = BA$ (b) $A^2 = B$
(c) $(AB)^T = \begin{bmatrix} 5 & 9 \\ 16 & 12 \end{bmatrix}$ (d) none of these
30. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction?
- (a) $A^n = 2^{n-1} A - (n-1) I$ (b) $A^n = nA - (n-1) I$
(c) $A^n = 2^{n-1} A + (n-1) I$ (d) $A^n = nA + (n-1) I$
- Type of matrices**
31. In the following, singular matrix is -
- (a) $\begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 6 \\ 4 & 12 \end{bmatrix}$
32. If $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & k & 5 \\ 4 & 2 & 1 \end{bmatrix}$ is a singular matrix, then k is equal to
- (a) -1 (b) 8
(c) 4 (d) -8
33. If $A = [a_{ij}]$ is a skew-symmetric matrix of order n , then $a_{ii} =$
- (a) 0 for some i (b) 0 for all $i = 1, 2, \dots, n$
(c) 1 for some i (d) 1 for all $i = 1, 2, \dots, n$



34. Matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is a -

- (a) diagonal matrix (b) upper triangular matrix
(c) skew-symmetric matrix (d) symmetric matrix

35. If A and B are square matrices of same order, then which of the following is skew-symmetric -

- (a) $\frac{A + A^T}{2}$ (b) $\frac{A^T + B^T}{2}$
(c) $\frac{A^T - B^T}{2}$ (d) $\frac{B - B^T}{2}$

36. If A is symmetric as well as skew symmetric matrix, then -

- (a) A is a diagonal matrix (b) A is a null matrix
(c) A is a unit matrix (d) A is a triangular matrix

37. If $A = \begin{bmatrix} -1 & 7 \\ 2 & 3 \end{bmatrix}$, then skew-symmetric part of A is -

- (a) $\begin{bmatrix} -1 & 9/2 \\ -9/2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} -1 & -9/2 \\ 9/2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$

Adjoint of matrix and its properties

38. If $A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 1 \\ 5 & 1 & 3 \end{bmatrix}$, then adj. A is equal to -

- (a) $\begin{bmatrix} 14 & -4 & -22 \\ -4 & -22 & 14 \\ -22 & 14 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} -14 & 4 & 22 \\ 4 & 22 & -14 \\ 22 & -14 & 4 \end{bmatrix}$
(c) $\begin{bmatrix} 14 & 4 & -22 \\ 4 & -22 & -14 \\ -22 & -14 & -4 \end{bmatrix}$ (d) none of these

39. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 0 & 4 \\ 2 & 6 & 7 \end{bmatrix}$ then adj A is equal to -

- (a) $\begin{bmatrix} -24 & 4 & 8 \\ 4 & 1 & 2 \\ 8 & 11 & -11 \end{bmatrix}$ (b) $\begin{bmatrix} -24 & 4 & 8 \\ 4 & 1 & 11 \\ 30 & -2 & -10 \end{bmatrix}$

- (c) $\begin{bmatrix} -24 & 4 & 8 \\ -27 & 1 & 11 \\ 30 & -2 & -10 \end{bmatrix}$ (d) none of these

40. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix}$, then A (adj A) equals -

- (a) $\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ (b) $-\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

- (c) $\begin{bmatrix} 0 & 0 & 9 \\ 0 & 9 & 0 \\ 9 & 0 & 0 \end{bmatrix}$ (d) none of these

41. If A is an invertible matrix of order n, then the determinant of Adj. A =

- (a) $|A|^n$ (b) $|A|^{n+1}$
(c) $|A|^{n-1}$ (d) $|A|^{n+2}$

Inverse of a matrix and Its properties

42. If A and B are invertible matrices of the order n, then $(AB)^{-1}$ is equal to

- (a) AB^{-1} (b) $A^{-1}B$
(c) $B^{-1}A^{-1}$ (d) $A^{-1}B^{-1}$

43. If $A^2 - A + I = 0$, then the inverse of A is

- (a) A (b) A + I
(c) I - A (d) A - I



44. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then $19A^{-1}$ is equal to

- (a) A' (b) $2A$
(c) $\frac{1}{2}A$ (d) A

45. $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}^{-1} =$

- (a) $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}$
(c) $\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & -5 \\ 7 & -6 \end{bmatrix}$

46. Inverse matrix of $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$ is -

- (a) $-\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (b) $-\frac{1}{8}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$
(c) $\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$

47. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $M = AB$, then M^{-1} is

equal to -

- (a) $\begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 1/6 \end{bmatrix}$
(c) $\begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 1/6 \end{bmatrix}$ (d) $\begin{bmatrix} 1/3 & -1/3 \\ -1/3 & 1/6 \end{bmatrix}$

48. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and X is a matrix such that $A = BX$, then X equals -

- (a) $\frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$
(c) $\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (d) none of these

49. Matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is not invertible if -

- (a) $\lambda = -15$ (b) $\lambda = -17$
(c) $\lambda = -16$ (d) $\lambda = -18$

50. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ then A^{-n} is equal to -

- (a) $\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ -n & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$ (d) none of these

51. $\begin{bmatrix} 1 & -\tan \theta/2 \\ \tan \theta/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta/2 \\ -\tan \theta/2 & 1 \end{bmatrix}^{-1}$ is equal to

- (a) $\begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$ (b) $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$
(c) $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ (d) none of these

Consistency of simultaneous Equations

52. The system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution if

- (a) $k \neq 0$ (b) $-1 < k < 1$
(c) $-2 < k < 2$ (d) $k = 0$



53. If the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ has no solution, then the values of λ and μ are

- (a) $\lambda = 3, \mu = 10$ (b) $\lambda = 3, \mu \neq 10$
(c) $\lambda \neq 3, \mu = 10$ (d) $\lambda \neq 3, \mu \neq 10$

54. Consider the system of equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$ if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0, \text{ then the system has}$$

- (a) more than two solutions
(b) only non trivial solutions
(c) no solution
(d) only trivial solution $(0, 0, 0)$.

55. Consider the system of linear equations

$$x_1 + 2x_2 + x_3 = 3$$

$$2x_1 + 3x_2 + x_3 = 3$$

$$3x_1 + 5x_2 + 2x_3 = 1$$

The system has

- (a) Infinite number of solutions
(b) Exactly 3 solutions
(c) A unique solution
(d) No solution

56. If the trivial solution is the only solution of the system of equations

$$x - ky + z = 0$$

$$kx + 3y - kz = 0$$

$$3x + y - z = 0$$

Then, the set of all values of k is

- (a) $\{2, -3\}$ (b) $R - \{2, -3\}$
(c) $R - \{2\}$ (d) $R - \{-3\}$

57. The number of values of k , for which the system of equations

$$(k + 1)x + 8y = 4k$$

$$kx + (k + 3)y = 3k - 1$$

has no solution, is

- (a) infinite (b) 1
(c) 2 (d) 3

58. If a, b, c are non-zero real numbers and if the system of equations

$$(a - 1)x = y + z,$$

$$(b - 1)y = z + x,$$

$$(c - 1)z = x + y,$$

has a non-trivial solution, then $ab + bc + ca$ equals:

- (a) $a + b + c$ (b) abc
(c) 1 (d) -1

59. With the help of matrices, the solution of the equations

$$3x + y + 2z = 3,$$

$$2x - 3y - z = -3,$$

$x + 2y + z = 4$ is given by

- (a) $x = 1, y = 2, z = -1$ (b) $x = -1, y = 2, z = 1$
(c) $x = 1, y = -2, z = -1$ (d) $x = -1, y = -2, z = 1$

60. Solution of

$$x + 3y - 2z = 0$$

$$2x - y + 4z = 0$$

$$x - 11y + 14z = 0$$

- (a) $\frac{x}{8} = \frac{y}{-10} = \frac{z}{7} = \lambda$ (b) $\frac{x}{-10} = \frac{y}{8} = \frac{z}{7} = \lambda$
(c) $\frac{x}{7} = \frac{y}{8} = \frac{z}{-10} = \lambda$ (d) None of these

Numerical Value Type Questions

61. Let $f(\theta) = \begin{vmatrix} \cos^2 \theta & \cos \theta \sin \theta & -\sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta & \cos \theta \\ \sin \theta & -\cos \theta & 0 \end{vmatrix}$ then $f\left(\frac{\pi}{6}\right) =$



62. If $f(x) = \tan x$ and A, B, C are the angles of

$$\Delta \begin{vmatrix} f(A) & f(\pi/4) & f(\pi/4) \\ f(\pi/4) & f(B) & f(\pi/4) \\ f(\pi/4) & f(\pi/4) & f(C) \end{vmatrix}$$

then is equal to

63. $\begin{vmatrix} 109 & 102 & 95 \\ 6 & 13 & 20 \\ 1 & -6 & -13 \end{vmatrix}$ is equal to

64. If $\omega (\neq 1)$ is a cube root of unity, then

$$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} =$$

65. If $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = k(xyz)$, then k is equal to

66. If $ax^3 + bx^2 + cx + d = \begin{vmatrix} 3x & x+1 & x-1 \\ x-3 & -2x & x+2 \\ x+3 & x-4 & 5x \end{vmatrix}$

be an identity in x , where a, b, c are constants, then the value of $-d$ is

67. If l, m, n are the $p^{\text{th}}, q^{\text{th}}$ and r^{th} term of a G.P. all positive, then

$$\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} \text{ equals}$$

68. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $A^2 - 4A - nI = 0$, then $-n$ is equal to -

69. If $\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$, then find the value of

$$x + \frac{y}{2} + \frac{z}{3}$$

70. If p and q are real so that the system of equations $px + 4y + z = 0$, $2y + 3z = 1$ and $3x - qz = -2$ has infinite

solutions then $\sqrt{q^2 - p^2}$ is equal to -

71. The system of equations $\lambda x + y + z = 1$, $x + \lambda y + z = \lambda$ and $x + y + \lambda z = \lambda^2$ have no solution. Then the value of λ^4 is

72. The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + kz = k^2$ have no solution if $-k$ equals

73. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$ and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

74. The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution is

75. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $\det(A^n - I) = 1 - \lambda^n$, $n \in \mathbb{N}$, then λ is equal to



EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. The system of linear equations

$$\begin{aligned} x + \lambda y - z &= 0 \\ \lambda x - y - z &= 0 \\ x + y - \lambda z &= 0 \end{aligned}$$
 has a non-trivial solution for :
 - (a) exactly one value of λ .
 - (b) exactly two values of λ .
 - (c) exactly three values of λ .
 - (d) infinitely many values of λ .
2. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \operatorname{adj} A = AA^T$, then $5a + b$ is equal to:
 - (a) 5
 - (b) 4
 - (c) 13
 - (d) -1
3. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2015} P$ is
 - (a) $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$
 - (b) $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$
 - (c) $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$
 - (d) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$
4. The number of distinct real roots of the equation,

$$\begin{vmatrix} \cos x & -\sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$
 in the intervals $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is:
 - (a) 4
 - (b) 3
 - (c) 2
 - (d) 1
5. Let A be a 3×3 matrix such that $A^2 - 5A + 7I = O$.

Statement - I : $A^{-1} = \frac{1}{7} (5I - A)$.

Statement - II : The Polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$. Then :

 - (a) Statement-I is true, but Statement-II is false.
 - (b) Statement-I is false, but Statement-II is true.
 - (c) Both the statements are true.
 - (d) Both the statements are false.
6. If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is :
 - (a) 2014
 - (b) -175
 - (c) 2016
 - (d) -25
7. If S is the set of distinct values of 'b' for which the following system of linear equations

$$\begin{aligned} x + y + z &= 1 \\ x + ay + z &= 1 \\ ax + by + z &= 0 \end{aligned}$$
 has no solution, then S is :
 - (a) an empty set
 - (b) an infinite set
 - (c) a finite set containing two or more elements
 - (d) a singleton
8. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$$
 then k is equal to:
 - (a) -z
 - (b) z
 - (c) -1
 - (d) 1



9. Let A be any 3×3 invertible matrix. Then which one of the following is **not** always true ? (2017)

- (a) $\text{adj } A = |A| \cdot A^{-1}$
 (b) $\text{adj } (\text{adj } A) = |A| \cdot A$
 (c) $\text{adj } (\text{adj } A) = |A|^2 \cdot (\text{adj } A)^{-1}$
 (d) $\text{adj } (\text{adj } (A)) = |A| \cdot (\text{adj } (A))^{-1}$

10. If $S = \left\{ x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\}$, then

$\sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right)$ is equal to : (2017)

- (a) $4 + 2\sqrt{3}$ (b) $-2 + \sqrt{3}$
 (c) $-2 - \sqrt{3}$ (d) $-4 - 2\sqrt{3}$

11. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then $\text{adj } (3A^2 + 12A)$ is equal to:

(2017)

- (a) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (b) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$
 (c) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (d) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

12. For two 3×3 matrices A and B , let $A + B = 2B'$ and $3A + 2B = I_3$, where B' is the transpose of B and I_3 is 3×3 identity matrix. Then : (2017)

- (a) $5A + 10B = 2I_3$ (b) $10A + 5B = 3I_3$
 (c) $B + 2A = I_3$ (d) $3A + 6B = 2I_3$

13. The number of real values of λ for which the system of linear equations

$$2x + 4y - \lambda z = 0$$

$$4x + \lambda y + 2z = 0$$

$$\lambda x + 2y + 2z = 0$$

has infinitely many solutions, is (2017/Online Set-1)

- (a) 0 (b) 1
 (c) 2 (d) 3

14. If the system of linear equations

$$x + ky + 3z = 0$$

$$3x + ky - 2z = 0$$

$$2x + 4y - 3z = 0$$

has a non-zero solution (x, y, z) , then $\frac{xz}{y^2}$ is equal to :

(2018)

- (a) 30 (b) -10
 (c) 10 (d) -30

15. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the

ordered pair (A, B) is equal to :

(2018)

- (a) (4, 5) (b) (-4, -5)
 (c) (-4, 3) (d) (-4, 5)

16. Let A be matrix such that $A \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$. Then A^2 equals : (2018/Online Set-1)

- (a) $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$ (b) $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$
 (c) $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$ (d) $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

17. If $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2 \sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then $\lim_{x \rightarrow 0} \frac{f'(x)}{x}$

(2018/Online Set-1)

- (a) does not exist
 (b) exists and is equal to 2
 (c) exists and is equal to 0
 (d) exists and is equal to -2



18. Let S be the set of all real values of k for which the system of linear equations

$$x + y + z = 2$$

$$2x + y - z = 3$$

$$3x + 2y + kz = 4$$

has a unique solution. Then S is :

(2018/Online Set-1)

- (a) an empty set (b) equal to $\{0\}$
(c) equal to \mathbb{R} (d) equal to $\mathbb{R} - \{0\}$
19. Suppose A is any 3×3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$.

If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to :

(2018/Online Set-2)

- (a) 8 (b) 7
(c) 13 (d) 12

20. If the system of linear equations

$$x + ay + z = 3$$

$$x + 2y + 2z = 6$$

$$x + 5y + 3z = b$$

has no solution, then :

(2018/Online Set-2)

- (a) $a = -1, b = 9$ (b) $a = -1, b \neq 9$
(c) $a \neq -1, b = 9$ (d) $a = 1, b \neq 9$

21. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the

elements of the first column of B is :

(2018/Online Set-3)

- (a) 210 (b) 211
(c) 231 (d) 251

22. The number of values of k for which the system of linear equations,

$$(k+2)x + 10y = k$$

$$kx + (k+3)y = k-1$$

has no solution, is :

(2018/Online Set-3)

- (a) 1 (b) 2
(c) 3 (d) infinitely many

23. The greatest value of $c \in \mathbb{R}$ for which the system of linear equations

$$x - cy - cz = 0$$

$$cx - y + cz = 0$$

$$cx + cy - z = 0$$

has a non-trivial solution, is :

(2019-04-08/Shift-1)

- (a) -1 (b) $\frac{1}{2}$
(c) 2 (d) 0

24. Let the numbers 2, b , c be in an A.P. and

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}. \text{ If } \det(A) \in [2, 16], \text{ then } c \text{ lies in the}$$

interval :

(2019-04-08/Shift-2)

- (a) $[2, 3)$ (b) $(2 + 2^{3/4}, 4)$
(c) $[4, 6]$ (d) $[3, 2 + 2^{3/4}]$

25. If the system of linear equations

$$x - 2y + kz = 1$$

$$2x + y + z = 2$$

$$3x - y - kz = 3$$

has a solution (x, y, z) , $z \neq 0$, then (x, y) lies on the straight line whose equation is :

(2019-04-08/Shift-2)

- (a) $3x - 4y - 1 = 0$ (b) $4x - 3y - 4 = 0$
(c) $4x - 3y - 1 = 0$ (d) $3x - 4y - 4 = 0$

26. If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$ then

the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is

(2019-04-09/Shift-1)

- (a) $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$



27. The total number of matrices $A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$

$(x, y \in \mathbb{R}, x \neq y)$ for which $A^T A = 3I_3$ is:

(2019-04-09/Shift-2)

- (a) 2 (b) 3
(c) 6 (d) 4

28. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z) , then

$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to (2019-04-09/Shift-2)

- (a) $\frac{3}{4}$ (b) $\frac{1}{2}$
(c) $-\frac{1}{4}$ (d) -4

29. If $\Delta_1 = \begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$ and

$$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}, x \neq 0$$

then for all $\Delta_1 = \theta \in \left(0, \frac{\pi}{2}\right)$: (2019-04-10/Shift-1)

- (a) $\Delta_1 - \Delta_2 = -2x^3$
(b) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$
(c) $\Delta_1 \times \Delta_2 = -2(x^3 + x - 1)$
(d) $\Delta_1 + \Delta_2 = -2x^3$

30. Let λ be a real number for which the system of linear equations:

$$x + y + z = 6$$

$$4x + \lambda y - \lambda z = \lambda - 2$$

$$3x + 2y - 4z = -5$$

has infinitely many solutions. Then λ is a root of the quadratic equation: (2019-04-10/Shift-2)

- (a) $\lambda^2 + 3\lambda - 4 = 0$ (b) $\lambda^2 - 3\lambda - 4 = 0$
(c) $\lambda^2 + \lambda - 6 = 0$ (d) $\lambda^2 - \lambda - 6 = 0$

31. The sum of the real roots of the equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0, \text{ is equal to: (2019-04-10/Shift-2)}$$

- (a) 6 (b) 0
(c) 1 (d) -4

32. If A is a symmetric matrix and B is a skew-symmetric

matrix such that $A+B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to :

(2019-04-12/Shift-1)

- (a) $\begin{bmatrix} -4 & -1 \\ -1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$
(c) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

33. If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3×3 matrix A ,

then the sum of all values of α for which $\det(A) + 1 = 0$, is (2019-04-12/Shift-1)

- (a) 0 (b) -1
(c) 1 (d) 2



34. A value of $\theta \in \left(0, \frac{\pi}{3}\right)$, for which

$$\begin{vmatrix} 1 + \cos^2 \theta & \sin^2 \theta & 4 \cos 6\theta \\ \cos^2 \theta & 1 + \sin^2 \theta & 4 \cos 6\theta \\ \cos^2 \theta & \sin^2 \theta & 1 + 4 \cos 6\theta \end{vmatrix} = 0 \text{ is } \underline{\hspace{2cm}}.$$

(2019-04-12/Shift-2)

- (a) $\frac{\pi}{9}$ (b) $\frac{\pi}{18}$
(c) $\frac{7\pi}{24}$ (d) $\frac{7\pi}{36}$

35. The system of linear equations

$$x + y + z = 2$$

$$2x + 3y + 2z = 5$$

$$2x + 3y + (a^2 - 1)z = a + 1 \quad (2019-01-09/Shift-1)$$

- (a) is inconsistent when $a = 4$
(b) has a unique solution for $|a| = \sqrt{3}$
(c) has infinitely many solutions for $a = 4$
(d) is inconsistent when $|a| = \sqrt{3}$

36. If $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$ is equal to (2019-01-09/Shift-1)

(a) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

37. If the system of linear equations

$$x - 4y + 7z = g$$

$$3y - 5z = h$$

$$-2x + 5y - 9z = k$$

is consistent, then : (2019-01-09/Shift-2)

- (a) $g + 2h + k = 0$ (b) $g + h + 2k = 0$
(c) $2g + h + k = 0$ (d) $g + h + k = 0$

38. If $A = \begin{bmatrix} e^t & e^{-t} \cos t & e^{-t} \sin t \\ e^t & -e^{-t} \cos t - e^{-t} \sin t & -e^{-t} \sin t + e^{-t} \cos t \\ e^t & 2e^{-t} \sin t & -2e^{-t} \cos t \end{bmatrix}$

then A is: (2019-01-09/Shift-2)

- (a) invertible for all $t \in \mathbb{R}$
(b) invertible only if $t = \pi$
(c) not invertible for any $t \in \mathbb{R}$
(d) invertible only if $t = \frac{\pi}{2}$

39. If the system of equations

$$x + y + z = 5$$

$$x + 2y + 3z = 9$$

$$x + 3y + \alpha z = \beta$$

has infinitely many solutions, then $\beta - \alpha$ equals:

(2019-01-10/Shift-1)

40. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$$x + 3y + 7z = 0$$

$$-x + 4y + 7z = 0$$

$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$ has a non-trivial solution, is (2019-01-10/Shift-2)

- (a) three (b) two
(c) four (d) one



41. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum

value of $\frac{\det(A)}{b}$ is: (2019-01-10/Shift-2)

- (a) $2\sqrt{3}$ (b) $-2\sqrt{3}$
(c) $-\sqrt{3}$ (d) $\sqrt{3}$

42. Let $a_1, a_2, a_3, \dots, a_{10}$ be in G.P. with $a_i > 0$ for $i = 1, 2, \dots, 10$ and S be the set of pairs (r, k) , $r, k \in \mathbb{N}$ (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S , is :

(2019-01-10/Shift-2)

- (a) 4 (b) infinitely many
(c) 2 (d) 10

43. If the system of linear equations

$$2x + 2y + 3z = a$$

$$3x - y + 5z = b$$

$$x - 3y + 2z = c$$

where, a, b, c are non-zero real numbers, has more than one solution, then :

(2019-01-11/Shift-1)

- (a) $b - c + a = 0$ (b) $b - c - a = 0$
(c) $a + b + c = 0$ (d) $b + c - a = 0$

44. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$$= (a+b+c)(x+a+b+c)^2, x \neq 0 \text{ and } a+b+c \neq 0,$$

then x is equal to : (2019-01-11/Shift-2)

- (a) abc (b) $-(a+b+c)$
(c) $2(a+b+c)$ (d) $-2(a+b+c)$

45. Let A and B be two invertible matrices of order 3×3 . If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$ then $\det(BA^{-1}B^T)$ is equal to : (2019-01-11/Shift-2)

- (a) $\frac{1}{4}$ (b) 1
(c) $\frac{1}{16}$ (d) 16

46. An ordered pair (α, β) for which the system of linear equations

$$(1+\alpha)x + \beta y + z = 2,$$

$$\alpha x + (1+\beta)y + z = 3,$$

$$\alpha x + \beta y + 2z = 2$$

has a unique solution, is: (2019-01-12/Shift-1)

- (a) $(2, 4)$ (b) $(-3, 1)$
(c) $(-4, 2)$ (d) $(1, -3)$

47. If $A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$; then for all

$$\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4} \right), \det(A) \text{ lies in the interval :}$$

(2019-01-12/Shift-2)

- (a) $\left(1, \frac{5}{2} \right]$ (b) $\left[\frac{5}{2}, 4 \right)$
(c) $\left(0, \frac{3}{2} \right]$ (d) $\left(\frac{3}{2}, 3 \right]$



48. The set of all values of λ for which the system of linear equations

$$x - 2y - 2z = \lambda x$$

$$x + 2y + z = \lambda y$$

$$-x - y = \lambda z$$

has a non-trivial solution : (2019-01-12/Shift-2)

- (a) is a singleton
- (b) contains exactly two elements
- (c) is an empty set
- (d) contains more than two elements

49. Let A be a 2×2 real matrix with entries from $\{0, 1\}$ and $|A| \neq 0$. Consider the following two statements :

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $\text{tr}(A) = 2$,

Where I_2 denotes 2×2 identity matrix and $\text{tr}(A)$ denotes the sum of the diagonal entries of A . Then :

(2020-09-02/Shift-1)

- (a) Both (P) and (Q) are false
- (b) (P) is true and (Q) is false
- (c) Both (P) and (Q) are true
- (d) (P) is false and (Q) is true

50. Let S be the set of all $\lambda \in R$ for which the system of linear equations

$$2x - y + 2z = 2$$

$$x - 2y + \lambda z = -4$$

$$x + \lambda y + z = 4$$

has no solution. Then the set S (2020-09-02/Shift-1)

- (a) is an empty set.
- (b) is a singleton.
- (c) contains more than two elements.
- (d) contains exactly two elements.

51. Let $A = \{X = (x, y, z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1\}$, where

$$P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}, \text{ then the set } A : (2020-09-02/Shift-2)$$

- (a) contains more than two elements
- (b) is a singleton.
- (c) contains exactly two elements
- (d) is an empty set.

52. Let $a, b, c \in R$ be all non-zero and satisfy

$$a^3 + b^3 + c^3 = 2 \text{ If the matrix } A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix} \text{ satisfies}$$

$A^T A = I$, then a value of abc can be :

(2020-09-02/Shift-2)

- (a) $\frac{2}{3}$
- (b) 3
- (c) $a - b$
- (d) $\frac{1}{3}$

53. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$,

then $B + C$ is equal to : (2020-09-03/Shift-1)

- (a) 1
- (b) -1
- (c) -3
- (d) 9

54. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then

a_{22} is equal to (2020-09-03/Shift-1)



55. Let A be a 3×3 matrix such that $\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$

and $B = \text{adj}(\text{adj } A)$. If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$ then the ordered pair, $(|\lambda|, \mu)$ is equal to : (2020-09-03/Shift-2)

(a) $\left(9, \frac{1}{81}\right)$ (b) $\left(9, \frac{1}{9}\right)$

(c) $\left(3, \frac{1}{81}\right)$ (d) $(3, 81)$

56. Let S be the set of all integer solutions, (x, y, z) , of the system of equations

$$x - 2y + 5z = 0$$

$$-2x + 4y + z = 0$$

$$-7x + 14y + 9z = 0$$

such that $15 \leq x^2 + y^2 + z^2 \leq 150$. Then, the number of elements in the set S is equal to

(2020-09-03/Shift-2)

57. If $A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

where $i = \sqrt{-1}$ then which one of the following is not true ? (2020-09-04/Shift-1)

(a) $a^2 - d^2 = 0$ (b) $a^2 - c^2 = 1$

(c) $0 \leq a^2 + b^2 \leq 1$ (d) $a^2 - b^2 = \frac{1}{2}$

58. If the system of equations

$$x - 2y + 3z = 9$$

$$2x + y + z = b$$

$$x - 7y + az = 24$$

has infinitely many solutions, then $a - b$ is equal to.....

(2020-09-04/Shift-1)

59. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1, b_2 and b_3 respectively.

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \text{ and } b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix},$$

If then the determinant of A is equal to:

(2020-09-04/Shift-2)

(a) 2 (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) 4

60. If the system of equations

$$x + y + z = 2$$

$$2x + 4y - z = 6$$

$$3x + 2y + \lambda z = \mu$$

has infinitely many solutions, then

(2020-09-04/Shift-2)

(a) $\lambda - 2\mu = -5$ (b) $2\lambda + \mu = 14$

(c) $\lambda + 2\mu = 14$ (d) $2\lambda - \mu = 5$

61. If the minimum and the maximum values of the function

$$f: \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \rightarrow \mathbb{R}, \text{ defined by}$$

$$f(\theta) = \begin{vmatrix} -\sin^2 \theta & -1 - \sin^2 \theta & 1 \\ -\cos^2 \theta & -1 - \cos^2 \theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

are m and M respectively, then the ordered pair (m, M) is equal to:

(2020-09-05/Shift-1)

(a) (0, 4) (b) (-4, 0)

(c) (-4, 4) (d) $(0, 2\sqrt{2})$



62. Let $\lambda \in \mathbb{R}$. The system of linear equations
- $$2x_1 - 4x_2 + \lambda x_3 = 1$$
- $$x_1 - 6x_2 + x_3 = 2$$
- $$\lambda x_1 - 10x_2 + 4x_3 = 3$$
- is inconsistent for: (2020-09-05/Shift-1)

- (a) exactly two values of λ
- (b) exactly one negative value of λ
- (c) every value of λ
- (d) exactly one positive value of λ

63. $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero

distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to:

(2020-09-05/Shift-2)

- (a) $y(a-b)$ (b) 0
- (c) $y(b-a)$ (d) $y(a-c)$

64. If the system of linear equations
- $$x + y + 3z = 0$$
- $$x + 3y + k^2z = 0$$
- $$3x + y + 3z = 0$$
- has a non-zero solution (x, y, z) for some $k \in \mathbb{R}$, then

$x + \left(\frac{y}{z}\right)$ is equal to: (2020-09-05/Shift-2)

(a) -9 (b) 9

(c) -3 (d) 3

65. The values of λ and μ for which the system of linear equations
- $$x + y + z = 2$$
- $$x + 2y + 3z = 5$$
- $$x + 3y + \lambda z = \mu$$
- has infinitely many solutions are, respectively:

(2020-09-06/Shift-1)

- (a) 6 and 8 (b) 5 and 8
- (c) 5 and 7 (d) 4 and 9

66. Let m and M be respectively the minimum and maximum

values of $\begin{vmatrix} \cos^2 x & 1 + \sin^2 x & \sin 2x \\ 1 + \cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1 + \sin 2x \end{vmatrix}$

Then the ordered pair (m, M) is equal to:

(2020-09-06/Shift-1)

- (a) $(-3, -1)$ (b) $(-4, -1)$
- (c) $(1, 3)$ (d) $(-3, 3)$

67. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$. If $B = A + A^4$, then $\det(B)$: (2020-09-06/Shift-2)

- (a) is one (b) lies in $(1, 2)$
- (c) lies in $(2, 3)$ (d) is zero

68. The sum of distinct values of λ for which the system of equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$, has non-zero solutions, is _____ (2020-09-06/Shift-2)

69. Let α be the root of the equation $x^2 + x + 1 = 0$ and the

matrix $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal

to, (2020-01-07/Shift-1)

- (a) A (b) A^2
- (c) A^3 (d) I_3



70. If the system of linear equations

$$2x + 2ay + az = 0$$

$$2x + 3by + bz = 0$$

$$2x + 4cy + c = 0$$

Where $a, b, c \in \mathbb{R}$ are non-zero and distinct; has non-zero solution, then (2020-01-07/Shift-1)

- (a) $a + b + c = 0$ (b) a, b, c are A.P.
(c) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. (d) a, b, c are in G.P.

71. $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)} a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is :

(2020-01-07/Shift-2)

- (a) $\frac{1}{9}$ (b) $\frac{1}{81}$
(c) $\frac{1}{3}$ (d) 3

72. If system of linear equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$3x + 2y + \lambda z = \mu$$

has more than two solutions, then $\mu - \lambda^2$ is equal to _____.

(2020-01-07/Shift-2)

73. For which of the following ordered pairs (μ, δ) , the system of linear equations

$$x + 2y + 3z = 1$$

$$3x + 4y + 5z = \mu$$

$$4x + 4y + 4z = \delta$$

is inconsistent?

(2020-01-08/Shift-1)

- (a) (4, 6) (b) (3, 4)
(c) (1, 0) (d) (4, 3)

74. The number of all 3×3 matrices A, with entries from the set $\{-1, 0, 1\}$ such that the sum of the diagonal elements of (AA^T) is 3, is _____. (2020-01-08/Shift-1)

75. If $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $10A^{-1}$ is equal to:

(2020-01-08/Shift-2)

- (a) $6I - A$ (b) $A - 6I$
(c) $4I - A$ (d) $A - 4I$

76. The system of linear equations

$$\lambda x + 2y + 2z = 5$$

$$2\lambda x + 3y + 5z = 8$$

$$4x + \lambda y + 6z = 10$$

has: (2020-01-08/Shift-2)

- (a) no solution when $\lambda = 2$
(b) infinitely many solutions when $\lambda = 2$
(c) no solution when $\lambda = 8$
(d) a unique solution when $\lambda = -8$

77. If for some α and β in \mathbb{R} , the intersection of the following three planes

$$x + 4y - 2z = 1$$

$$x + 7y - 5z = \beta$$

$$x + 5y + \alpha z = 5$$

is a line in \mathbb{R}^3 , then $\alpha + \beta$ is equal to:

(2020-01-09/Shift-1)

- (a) 0 (b) 10
(c) -10 (d) 2

78. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj } A$ and $C = 3A$,

then $\frac{|\text{adj } B|}{|C|}$ is equal to: (2020-01-09/Shift-1)

- (a) 16 (b) 2
(c) 8 (d) 72



79. Let $a - 2b + c = 1$. If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then:

(2020-01-09/Shift-2)

- (a) $f(-50) = 501$ (b) $f(-50) = 10$
(c) $f(50) = 1$ (d) $f(50) = -501$

80. The following system of linear equations

$$7x + 6y - 2z = 0,$$

$$3x + 4y + 2z = 0,$$

$$x - 2y - 6z = 0,$$

(2020-01-09/Shift-2)

- (a) infinitely many solutions, (x, y, z) satisfying $y = 2z$
(b) infinitely many solutions (x, y, z) satisfying $x = 2z$
(c) no solution
(d) only the trivial solution

81. Let $A = [a_{ij}]$ be a 3×3 matrix,

$$\text{where } a_{ij} = \begin{cases} 1, & \text{if } i = j \\ -x, & \text{if } |i - j| = 1 \\ 2x + 1, & \text{otherwise} \end{cases}$$

Let a function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \det(A)$. Then the sum of maximum and minimum values of f on \mathbb{R} is equal to:

(2021-07-20/Shift-1)

- (a) $\frac{20}{27}$ (b) $-\frac{88}{27}$
(c) $-\frac{20}{27}$ (d) $\frac{88}{27}$

82. Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in \mathbb{R}$ be written as $P + Q$, where P is a symmetric matrix and Q is a skew-symmetric matrix. If $\det(Q) = 9$, then the modulus of the sum of all possible values of determinant of P is equal to:

(2021-07-20/Shift-1)

- (a) 24 (b) 18
(c) 45 (d) 36

83. Let a, b, c, d be in arithmetic progression with common difference λ .

If $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$, then value of λ^2 is equal

to _____.

(2021-07-20/Shift-1)

84. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where

I is an identity matrix of order 3×3 . If $B = [b_{ij}]$, then b_{13} is equal to _____.

(2021-07-20/Shift-1)

85. The value of $k \in \mathbb{R}$, for which the following system of linear equations

$$3x - y + 4z = 3,$$

$$x + 2y - 3z = -2,$$

$$6x + 5y + kz = -3,$$

has infinitely many solutions, is: (2021-07-20/Shift-2)

- (a) 3 (b) -3
(c) 5 (d) -5

86. Let $A = \{a_{ij}\}$ be a 3×3 matrix,

$$\text{where } a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j \end{cases} \text{ then } \det(3\text{Adj}(2A^{-1})) \text{ is}$$

equal to _____ ?

(2021-07-20/Shift-2)

87. Let $S = \left\{ n \in \mathbb{N} \mid \begin{vmatrix} 0 & i \\ 1 & 0 \end{vmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a, b, c, d \in \mathbb{R} \right\}$,

where $i = \sqrt{-1}$. Then the number of 2-digit numbers in the set S is _____ ? (2021-07-25/Shift-1)



88. The values of a and b , for which the system of equations

$$2x + 3y + 6z = 8$$

$$x + 2y + az = 5$$

$$3x + 5y + 9z = b$$

Has no solution, are ? (2021-07-25/Shift-1)

- (a) $a = 3, b = 13$ (b) $a \neq 3, b \neq 13$
(c) $a \neq 3, b = 3$ (d) $a = 3, b \neq 13$

89. $f(x) = \begin{vmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix}, x \in [0, \pi]$

Then the maximum value of $f(x)$ is equal to _____.

(2021-07-27/Shift-1)

90. For real numbers α and β consider the following system of linear equations:

$$x + y - z = 2, x + 2y + \alpha z = 1, 2x - y + z = \beta.$$

If the system has infinite solutions, then $\alpha + \beta$ is equal to _____.

(2021-07-27/Shift-1)

91. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in \mathbb{R}$, I is a 2×2 identity matrix, then $4(\alpha - \beta)$ is equal to:

(2021-07-27/Shift-1)

- (a) 5 (b) 4
(c) 2 (d) $\frac{8}{3}$

92. Let A and B be two 3×3 real matrices such that $(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and $A^3 B^2 = A^2 B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to:

(2021-07-27/Shift-2)

- (a) 0 (b) 2
(c) 1 (d) 4

93. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + \dots + A^{20}$, then

the sum of the all the elements of the matrix M is equal to _____.

(2021-07-27/Shift-2)

94. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3×3 matrices B

with entries from the set $\{1, 2, 3, 4, 5\}$ and satisfying $AB = BA$ is _____.

(2021-07-22/Shift-2)

95. Let $A = [a_{ij}]$ be a real matrix of order 3×3 such that $a_{i1} + a_{i2} + a_{i3} = 1$, for $i = 1, 2, 3$. Then, the sum of all the entries of the matrix A^3 is equal to:

(2021-07-22/Shift-2)

- (a) 1 (b) 3
(c) 2 (d) 9

96. The value of λ and μ such that the system of equations $x + y + z = 6$, $3x + 5y + 5z = 26$, $x + 2y + \lambda z = \mu$ has no solution, are:

(2021-07-22/Shift-2)

- (a) $\lambda = 3, \mu \neq 10$ (b) $\lambda \neq 2, \mu = 10$
(c) $\lambda = 3, \mu = 5$ (d) $\lambda = 2, \mu \neq 10$

97. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is:

(2021-07-25/Shift-2)

- (a) 1 (b) 2
(c) 3 (d) 4

98. If $P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$, then P^{50} is: (2021-07-25/Shift-2)

- (a) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$



99. Consider the system of linear equations

$$-x + y + 2z = 0$$

$$3x - ay + 5z = 1$$

$$2x - 2y - az = 7$$

Let S_1 be the set of all $a \in \mathbb{R}$ for which the system is inconsistent and S_2 be the set of all $a \in \mathbb{R}$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then (2021-09-01/Shift-2)

(a) $n(S_1) = 0, n(S_2) = 2$

(b) $n(S_1) = 2, n(S_2) = 2$

(c) $n(S_1) = 2, n(S_2) = 0$

(d) $n(S_1) = 1, n(S_2) = 0$

100. Let $J_{n,m} = \int_0^{1/2} \frac{x^n}{x^m - 1} dx, \forall n > m \text{ and } n, m \in \mathbb{N}$. Consider

a matrix $A = [a_{ij}]_{3 \times 3}$ where $a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i \leq j \\ 0, & i > j \end{cases}$.

Then $|\text{adj} A^{-1}|$ is (2021-09-01/Shift-2)

(a) $(15)^2 \times 2^{34}$ (b) $(105)^2 \times 2^{38}$

(c) $(15)^2 \times 2^{42}$ (d) $(105)^2 \times 2^{36}$

101. Two fair dice are thrown. The number on them are taken as λ and μ and a system of linear equations

$$x + y + z = 5$$

$$x + 2y + 3z = \mu$$

$$x + 3y + \lambda z = 1$$

Is constructed. If p is true probability that the system has a unique solution and q is the probability that the system has no solution, then: (2021-08-26/Shift-2)

(a) $p = \frac{5}{6}$ and $q = \frac{5}{36}$ (b) $p = \frac{1}{6}$ and $q = \frac{1}{36}$

(c) $p = \frac{1}{6}$ and $q = \frac{5}{36}$ (d) $p = \frac{5}{6}$ and $q = \frac{1}{36}$

102. Let A be a 3×3 real matrix.

If $\det(2\text{Adj}(2\text{Adj}(\text{Adj}(2A)))) = 2^{41}$, then the value of $\det(A^2)$ equals (2021-08-26/Shift-2)

103. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to: (2021-08-26/Shift-2)

(a) A^5 (b) A^6

(c) $A^6 - A$ (d) $A^5 - A$

104. If the matrix $A = \begin{pmatrix} 0 & 2 \\ k & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of k is: (2021-08-27/Shift-1)

(a) 1 (b) -1

(c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

105. If the system of linear equations

$$2x + y - z = 3$$

$$x - y - z = \alpha$$

$$3x + 3y + \beta z = 3$$

has infinitely many solutions, then $\alpha + \beta - \alpha\beta$ is equal to (2021-08-27/Shift-1)

106. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations,

$$(1 + \cos^2 \theta)x + (\sin^2 \theta)y + (4 \sin 3\theta)z = 0$$

$$(\cos^2 \theta)x + (1 + \sin^2 \theta)y + (4 \sin 3\theta)z = 0$$

$$(\cos^2 \theta)x + (\sin^2 \theta)y + (1 + 4 \sin 3\theta)z = 0$$

Has a non-trivial solution, then the value of θ is: (2021-08-26/Shift-1)

(a) $\frac{4\pi}{9}$ (b) $\frac{\pi}{18}$

(c) $\frac{5\pi}{18}$ (d) $\frac{7\pi}{18}$



107. If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -2 & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$ and

$Q = A^T B A$, then the inverse of the matrix $A Q^{2021} A^T$ is equal to : (2021-08-26/Shift-1)

(a) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

108. Let $[\lambda]$ be the greatest integer less than or equal to λ . The set of all value of λ for which the system of linear equations $x + y + z = 4$, $3x + 2y + 5z = 3$, $9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is (2021-08-27/Shift-2)

- (a) $(-\infty, -9) \cup [-8, \infty)$ (b) $(-\infty, -9) \cup (-9, \infty)$
(c) $[-9, -8]$ (d) \mathbb{R}

109. Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where $[x]$ denotes

the greatest integer less than or equal to x . If $\det(A) = 192$, then the set of values of x is in the interval:

(2021-08-27/Shift-2)

- (a) $[62, 63]$ (b) $[60, 61]$
(c) $[68, 69]$ (d) $[65, 66]$

110. If the following system of linear equations

$$2x + y + z = 5$$

$$x - y + z = 3$$

$$x + y + az = b$$

Has no solution, then ? (2021-08-31/Shift-1)

(a) $a \neq \frac{1}{3}, b = \frac{7}{3}$ (b) $a \neq -\frac{1}{3}, b = \frac{7}{3}$

(c) $a = \frac{1}{3}, b \neq \frac{7}{3}$ (d) $a = -\frac{1}{3}, b \neq \frac{7}{3}$

111. The number of elements in the set

$$\left\{ A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}; a, b, d \in \{-1, 0, 1\} \text{ and } (I - A)^3 = I - A^3 \right\},$$

where I is 2×2 identity matrix, is ____

(2021-08-31/Shift-2)

112. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$$x + (\cos \gamma)y + (\cos \beta)z = 0$$

$$(\cos \gamma)x + y + (\cos \alpha)z = 0$$

$$(\cos \beta)x + (\cos \alpha)y + z = 0$$

has: (2021-08-31/Shift-2)

- (a) exactly two solutions (b) a unique solution
(c) no solution (d) infinitely many solutions

113. Let A be the set of all points (α, β) such that the area of triangle formed by the points $(5, 6)$, $(3, 2)$ and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A , is:

(2021-08-31/Shift-2)

(a) $\frac{16}{\sqrt{5}}$ (b) $\frac{12}{\sqrt{5}}$

(c) $\frac{8}{\sqrt{5}}$ (d) $\frac{4}{\sqrt{5}}$



114. Consider the following system of equations:

$$x + 2y - 3z = a$$

$$2x + 6y - 11z = b$$

$$x - 2y + 7z = c,$$

where a, b and c are real constants. Then the system of equations: (2021-02-26/Shift-2)

- (a) has a unique solution when $5a = 2b + c$
- (b) has no solution for all a, b and c
- (c) has infinite number of solutions when $5a = 2b + c$
- (d) has a unique solution for all a, b and c

115. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation

$$A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ for some real numbers } \alpha$$

and β , then $\beta - \alpha$ is equal to _____.

(2021-02-26/Shift-2)

116. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is :

(2021-02-26/Shift-1)

- (a) 12
- (b) 4
- (c) 6
- (d) 1

117. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is :

(2021-02-26/Shift-1)

- (a) $(a+2)(a+3)(a+4)$
- (b) 0
- (c) -2
- (d) $(a+1)(a+2)(a+3)$

118. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and

$(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to _____. (2021-02-25/Shift-1)

119. Let α, β and γ are real numbers such that $\alpha + \beta + \gamma = 0$. If $\alpha^2 + \beta^2 + \gamma^2 = 1$, then the value of $\alpha\beta + \beta\gamma + \gamma\alpha$ is _____. (2021-02-25/Shift-1)

120. If the system of equations

$$kx + y + 2z = 1$$

$$3x - y - 2z = 2$$

$$-2x - 2y - 4z = 3$$

has infinitely many solutions, then k is equal to _____. (2021-02-25/Shift-1)

121. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is: (2021-02-25/Shift-2)

- (a) 3
- (b) 1
- (c) 4
- (d) 2

122. The following system of linear equations

$$2x + 3y + 2z = 9$$

$$3x + 2y + 2z = 9$$

$$x - y + 4z = 8$$

(2021-02-25/Shift-2)

- (a) does not have any solution
- (b) has infinitely many solutions
- (c) has a unique solution
- (d) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

123. Let A be a 3×3 matrix with $\det(A) = 4$. Let R_i denote the i^{th} row of A . If a matrix B is obtained by performing the operation $R_2 \rightarrow 2R_2 + 5R_3$ on $2A$, then $\det(B)$ is equal to: (2021-02-25/Shift-2)

- (a) 80
- (b) 128
- (c) 64
- (d) 16



124. For the system of linear equations:

$$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in \mathbb{R}$$

consider the following statements :

- (A) The system has unique solution if $k \neq 2, k \neq -2$.
 (B) The system has unique solution if $k = -2$.
 (C) The system has unique solution if $k = 2$.
 (D) The system has no-solution if $k = 2$.
 (E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct ?

(2021-02-24/Shift-2)

- (a) (B) and (E) only (b) (A) and (E) only
 (c) (A) and (D) only (d) (C) and (D) only

125. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2 B^2 - B^2 A^2)X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has

(2021-02-24/Shift-2)

- (a) a unique solution (b) exactly two solutions
 (c) no solution (d) infinitely many solutions

126. The system of linear equations

$$3x - 2y - kz = 10$$

$$2x - 4y - 2z = 6$$

$$x + 2y - z = 5m$$

is inconsistent if:

(2021-02-24/Shift-1)

- (a) $k = 3, m = \frac{4}{5}$ (b) $k \neq 3, m \in \mathbb{R}$
 (c) $k \neq 3, m \neq \frac{4}{5}$ (d) $k = 3, m \neq \frac{4}{5}$

127. Let M be any 3×3 matrix with entries from the set $\{0, 1, 2\}$. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is _____.

(2021-02-24/Shift-1)

128. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose

$Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in \mathbb{R}$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to _____.

(2021-02-24/Shift-1)

129. The solutions of the equation

$$\begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0, (0 < x < \pi), \text{ are :}$$

(2021-03-18/Shift-1)

- (a) $\frac{7\pi}{12}, \frac{11\pi}{12}$ (b) $\frac{\pi}{12}, \frac{\pi}{6}$
 (c) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (d) $\frac{\pi}{6}, \frac{5\pi}{6}$

130. Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$.

If $T_r(A)$ denotes the sum of all diagonal elements of the matrix A, then $T_r(A) - T_r(B)$ has value equal to :

(2021-03-18/Shift-1)

- (a) 3 (b) 1
 (c) 2 (d) 0

131. Let α, β, γ be the real roots of the equation $x^3 + ax^2 + bx + c = 0$, ($a, b, c \in \mathbb{R}$ and $a, b \neq 0$). If the system of equation (in u, v, w) given by $\alpha u + \beta v + \gamma w = 0$; $\beta u + \gamma v + \alpha w = 0$;

$\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value

of $\frac{a^2}{b}$ is :

(2021-03-18/Shift-1)

- (a) 5 (b) 1
 (c) 3 (d) 0



132. Let the system of linear equations

$$4x + \lambda y + 2z = 0$$

$$2x - y + z = 0$$

$$\mu x + 2y + 3z = 0, \lambda, \mu \in \mathbb{R}.$$

has a non-trivial solution. Then which of the following is true ? (2021-03-18/Shift-2)

- (a) $\lambda = 3, \mu \in \mathbb{R}$ (b) $\lambda = 2, \mu \in \mathbb{R}$
(c) $\mu = -6, \lambda \in \mathbb{R}$ (d) $\mu = 6, \lambda \in \mathbb{R}$

133. Define a relation R over a class of $n \times n$ real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$ ".

Then which of the following is true?

(2021-03-18/Shift-2)

- (a) R is reflexive, symmetric but not transitive
(b) R is an equivalence relation
(c) R is symmetric, transitive but not reflexive,
(d) R is reflexive, transitive but not symmetric

134. Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$.

Then the value of $n \in \mathbb{N}$ for which $P^n = 5I - 8P$ is equal to _____. (2021-03-18/Shift-2)

135. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, i = \sqrt{-1}$. Then, the system of linear

equations $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has: (2021-03-16/Shift-1)

- (a) A unique solution
(b) Exactly two solutions
(c) Infinitely many solutions
(d) No solution

136. The total number of 3×3 matrices A having entries from the set $\{0, 1, 2, 3\}$ such that the sum of all the diagonal entries of AA^T is 9, is equal to _____.

(2021-03-16/Shift-1)

137. Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$

where $\omega = \frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order

3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to _____.

(2021-03-16/Shift-1)

138. The maximum value of

$$f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in \mathbb{R} \text{ is}$$

(2021-03-16/Shift-2)

- (a) $\sqrt{5}$ (b) 5
(c) $\frac{3}{4}$ (d) $\sqrt{7}$

139. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with

real numbers such that $A = XB$, where $X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$,

and $k \in \mathbb{R}$. If $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and

$(k^2 + 1)b_2^2 \neq -2b_1b_2$ then the value of k is _____.

(2021-03-16/Shift-2)

140. The system of equations $kx + y + z = 1, x + ky + z = k$ and $x + y + zx = k^2$ has no solution is k is equal to :

(2021-03-17/Shift-1)

- (a) 1 (b) -2
(c) -1 (d) 0



141. If $A = \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix}$ and $\det \left(A^2 - \frac{1}{2}I \right) = 0$, then a possible value of α is : (2021-03-17/Shift-1)
- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$
142. If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det(A^{10} - (\text{Adj}(2A))^{10})$ is equal to (2021-03-17/Shift-1)
143. If x, y, z are in arithmetic progression with common difference d , $x \neq 3d$, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is : (2021-03-17/Shift-2)
- (a) 36 (b) 12
(c) 6 (d) 72
144. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that $AB = B$ and $a + d = 2021$, then the value of $ad - bc$ is equal to (2021-03-17/Shift-2)
145. If $1, \log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in arithmetic progression for a real number x , then the value of the determinant $\begin{vmatrix} 2\left(x - \frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ is equal to : (2021-03-17/Shift-2)



EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. If $A = \text{diag}(d_1, d_2, d_3, \dots, d_n)$, then A^n is equal to

(a) $\text{diag}(d_1^{n-1}, d_2^{n-1}, d_3^{n-1}, \dots, d_n^{n-1})$

(b) $\text{diag}(d_1^n, d_2^n, d_3^n, \dots, d_n^n)$

(c) A

(d) none of these

2. If $f(x)$, $g(x)$ and $h(x)$ are three polynomials of degree 2, then

$$\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix} \text{ is a polynomial of degree}$$

(a) 2

(b) 3

(c) 4

(d) none of these

3. If n is not a multiple of 3 and 1, ω , ω^2 are the cube roots of unity, then

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix} \text{ is equal to}$$

(a) 0

(b) ω

(c) ω^2

(d) 1

4. $D = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is :

(a) -1

(b) 0

(c) 1

(d) none of these

5. If $\begin{vmatrix} x^2 - 2x + 3 & 7x + 2 & x + 4 \\ 2x + 7 & x^2 - x + 2 & 3x \\ 3 & 2x - 1 & x^2 - 4x + 7 \end{vmatrix} =$

$ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$ the value of g is

(a) 2

(b) 1

(c) -2

(d) none of these

6. Which one of the following is correct ? If A is non-singular matrix, then, :

(a) $\det(A^{-1}) = \det(A)$ (b) $\det(A^{-1}) = \frac{1}{\det(A)}$

(c) $\det(A^{-1}) = 1$

(d) none of these

7. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then the value of $|A^T A^{-1}|$ is

(a) $\cos 4x$

(b) $\sec^2 x$

(c) $-\cos 4x$

(d) 1

8. For what value of x , the matrix

$$\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix} \text{ is singular.}$$

(a) $x = 1, 2$

(b) $x = 0, 2$

(c) $x = 0, 1$

(d) $x = 0, 3$

9. If m is a positive integer and

$$\Delta_r = \begin{vmatrix} 2r-1 & {}^m C_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$$

Then the value of $\sum_{r=0}^m \Delta_r$ is

(a) 0

(b) $m^2 - 1$

(c) 2^m

(d) $2^m \sin^2(2^m)$

10. Let $D_r = \begin{vmatrix} a & 2^r & 2^{16}-1 \\ b & 3(4^r) & 2(4^{16}-1) \\ c & 7(8^r) & 4(8^{16}-1) \end{vmatrix}$, then the value of

$$\sum_{k=1}^{16} D_k \text{ is}$$

(a) 0

(b) $a + b + c$

(c) $ab + bc + ca$

(d) none of these



11. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, then $x =$

- (a) 3 (b) 5
(c) 2 (d) 4

12. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$ then x is equal to

- (a) 9 (b) -9
(c) 0 (d) None of these

13. $\Delta = \begin{vmatrix} p & 2-i & i+1 \\ 2+i & q & 3+i \\ 1-i & 3-i & r \end{vmatrix}$ is always (where $p, q, r \in \mathbb{R}$)

- (a) real (b) imaginary
(c) zero (d) none of these

14. If $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$, then

- (a) $\Delta = (a-b)(b-c)(c-a)$
(b) a, b, c are in G.P.
(c) b, c, a are in G.P.
(d) a, c, b are in G.P.

15. If $E(\theta) = \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix}$ and θ and ϕ differ by an

- odd multiple of $\pi/2$, then $E(\theta) \cdot E(\phi)$ is a
(a) Null matrix (b) Unit matrix
(c) Diagonal matrix (d) none of these.

16. If $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N+1 \end{vmatrix}$, then $\sum_{n=1}^N U_n$ is equal to

- (a) $2 \sum_{n=1}^N n$ (b) $2 \sum_{n=1}^N n^2$
(c) $\frac{1}{2} \sum_{n=1}^N n^2$ (d) 0

17. If $a^2 + b^2 + c^2 = -2$ and

$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Then $f(x)$ is a polynomial of degree :

- (a) 2 (b) 3
(c) 0 (d) 1

18. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$$G(y) = \begin{bmatrix} \cos y & 0 & \sin y \\ 0 & 1 & 0 \\ -\sin y & 0 & \cos y \end{bmatrix}$$
 then $[F(x)G(y)]^{-1}$ is equal to

- (a) $F(-x)G(-y)$ (b) $F(x^{-1})G(y^{-1})$
(c) $G(-y)F(-x)$ (d) $G(y^{-1})F(x^{-1})$

19. If the system of linear equations

$$x + 2ay + az = 0$$

$$x + 3by + bz = 0$$

$$x + 4cy + cz = 0$$

has a non-zero solution, then a, b, c :

- (a) are in AP (b) are in GP
(c) are in HP (d) satisfy $a + 2b + 3c = 0$

20. The system of equations

$$\alpha x + y + z = \alpha - 1$$

$$x + \alpha y + z = \alpha - 1$$

$$x + y + \alpha z = \alpha - 1$$

has no solution, if α is :

- (a) 1 (b) not -2
(c) either -2 or 1 (d) -2

21. The system of equation $-2x + y + z = 1$,

$$x - 2y + z = -2, x + y + \lambda z = 4$$
 will have no solution if

- (a) $\lambda = -2$ (b) $\lambda = -1$
(c) $\lambda = 3$ (d) none of these



22. The system of equations $x + 2y + 3z = 4$, $2x + 3y + 4z = 5$, $3x + 4y + 5z = 6$ has

(a) Infinitely many solutions
(b) No solution
(c) A unique solution
(d) None of these

23. If $a, b, c > 0$ & $x, y, z \in \mathbb{R}$ then the determinant

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^y + b^{-y})^2 & (b^y - b^{-y})^2 & 1 \\ (c^z + c^{-z})^2 & (c^z - c^{-z})^2 & 1 \end{vmatrix} =$$

(a) $a^x b^y c^z$ (b) $a^{-x} b^{-y} c^{-z}$
(c) $a^{2x} b^{2y} c^{2z}$ (d) zero

24. If $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$, then the value of $\frac{p}{x} + \frac{q}{y} + \frac{r}{z}$ is

(a) 0 (b) 1
(c) 2 (d) $4pqr$

25. Suppose $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and

$$D' = \begin{vmatrix} a_1 + pb_1 & b_1 + qc_1 & c_1 + ra_1 \\ a_2 + pb_2 & b_2 + qc_2 & c_2 + ra_2 \\ a_3 + pb_3 & b_3 + qc_3 & c_3 + ra_3 \end{vmatrix}. \text{ Then}$$

(a) $D' = D$ (b) $D' = D(1 - pqr)$
(c) $D' = D(1 + p + q + r)$ (d) $D' = D(1 + pqr)$

26. Let $ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h$

$$= \begin{vmatrix} (x+1) & (x^2+2) & (x^2+x) \\ (x^2+x) & (x+1) & (x^2+2) \\ (x^2+2) & (x^2+x) & x+1 \end{vmatrix}. \text{ Then}$$

(a) $g = 3$ and $h = -5$ (b) $g = -3$ and $h = -5$
(c) $g = -3$ and $h = -9$ (d) None of these

Objective Questions II

[One or more than one correct option]

27. If $\begin{vmatrix} x^2 + x & x+1 & x-2 \\ 2x^2 + 3x - 1 & 3x & 3x-3 \\ x^2 + 2x + 3 & 2x-1 & 2x-1 \end{vmatrix} = Ax + B$, where A and B

are constants, then

(a) $A + B = 12$ (b) $A - B = 36$
(c) $A^2 + B^2 = 720$ (d) $A + 2B = 0$

28. Let A be a symmetric matrix such that $A^5 = O$ and $B = I + A + A^2 + A^3 + A^4$, then B is

(a) symmetric (b) singular
(c) non-singular (d) skew symmetric

29. The determinant $\Delta = \begin{vmatrix} a^2 + x^2 & ab & ac \\ ab & b^2 + x^2 & bc \\ ac & bc & c^2 + x^2 \end{vmatrix}$ is

divisible by

(a) x (b) x^2
(c) x^3 (d) x^4

30. If $f(\theta) = \begin{vmatrix} 1 & 1 & -1 \\ 1 & e^{i\theta} & 1 \\ 1 & -1 & -e^{-i\theta} \end{vmatrix}$ then

(a) $\int_{-\pi/2}^{\pi/2} f(\theta) d\theta = 2 \int_0^{\pi/2} f(\theta) d\theta$

(b) $f(\theta)$ is purely real
(c) $f(\pi/2) = 2$
(d) None of these

31. The value of the determinant

$$\begin{vmatrix} \sqrt{6} & 2i & 3 + \sqrt{6} \\ \sqrt{12} & \sqrt{3} + \sqrt{8}i & 3\sqrt{2} + \sqrt{6}i \\ \sqrt{18} & \sqrt{2} + \sqrt{12}i & \sqrt{27} + 2i \end{vmatrix}, \text{ where } i = \sqrt{-1}, \text{ is}$$

(a) complex number (b) real number
(c) irrational number (d) rational number



32. If $a > b > c$ and the system of equations $ax + by + cz = 0$, $bx + cy + az = 0$, $cx + ay + bz = 0$ has a non trivial solution, then both the roots of the quadratic equation $at^2 + bt + c = 0$ are
- (a) real (b) of opposite sign
(c) positive (d) complex
33. If $\Delta = \begin{vmatrix} e^x & \sin x & 1 \\ \cos x & \log_e(1+x^2) & 1 \\ x & x^2 & 1 \end{vmatrix} = a + bx + cx^2 \dots$ then
- (a) $a = 0$ (b) $a = 1$
(c) $b = -1$ (d) $b = -2$
34. If $f(x)$ and $g(x)$ are functions such that $f(x+y) = f(x)g(y) + g(x)f(y)$, then
- $\begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha + \theta) \\ f(\beta) & g(\beta) & f(\beta + \theta) \\ f(\gamma) & g(\gamma) & f(\gamma + \theta) \end{vmatrix}$ is independent of
- (a) α (b) β
(c) γ (d) θ
35. $\begin{vmatrix} 1 & a & a^2 \\ \cos(xz - yz) & \cos xz & \cos(xz - yz) \\ \sin(xz - yz) & \sin xz & \sin(xz - yz) \end{vmatrix}$ depends on
- (a) x (b) y
(c) z (d) a
36. If $x \in \mathbb{N}$ and ${}^x C_i$, ${}^{x^2} C_i$ and ${}^{x^3} C_i$, ($i = 1, 2, 3$) are binomial coefficients, then
- $12 \begin{vmatrix} {}^x C_1 & {}^x C_2 & {}^x C_3 \\ {}^{x^2} C_1 & {}^{x^2} C_2 & {}^{x^2} C_3 \\ {}^{x^3} C_1 & {}^{x^3} C_2 & {}^{x^3} C_3 \end{vmatrix}$ is divisible by
- (a) x^3 (b) x^6
(c) x^9 (d) x^{12}
37. The digits A, B, C are such that the three digit numbers A 88, 6B8, 86C are divisible by 72, then the determinant
- $\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$ is divisible by
- (a) 72 (b) 144
(c) 288 (d) 216
38. Let $f(a, b) = \begin{vmatrix} a & a^2 & 0 \\ 1 & (2a+b) & (a+b)^2 \\ 0 & 1 & (2a+3b) \end{vmatrix}$, then
- (a) $(a+b)$ is a factor of $f(a, b)$
(b) $(a+2b)$ is a factor of $f(a, b)$
(c) $(2a+b)$ is a factor of $f(a, b)$
(d) a is a factor of $f(a, b)$
39. a, b, c are non-zero real numbers. Then
- $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$, if
- (a) $\frac{1}{a} + \frac{1}{b\omega} + \frac{1}{c\omega^2} = 0$ (b) $\frac{1}{a} + \frac{1}{b\omega^2} + \frac{1}{c\omega} = 0$
(c) $\frac{1}{a\omega} + \frac{1}{b\omega^2} + \frac{1}{c} = 0$ (d) none of these
40. Let $f(x) = \begin{vmatrix} 1/x & \ln x & x^n \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$, then $\frac{d^n}{dx^n} f(x)$ at $x = 1$ is
- (a) independent of a (b) independent of n
(c) independent of a and n (d) zero
41. If $a^2 + b^2 + c^2 = 1$, then
- $\begin{vmatrix} a^2 + (b^2 + c^2)\cos\phi & ab(1 - \cos\phi) & ac(1 - \cos\phi) \\ ba(1 - \cos\phi) & b^2 + (c^2 + a^2)\cos\phi & bc(1 - \cos\phi) \\ ca(1 - \cos\phi) & cb(1 - \cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$
- is independent of
- (a) a (b) b
(c) c (d) ϕ



42. Let A and B be two matrices different from I such that $AB = BA$ and $A^n - B^n$ is invertible for some positive integer n . If $A^n - B^n = A^{n+1} - B^{n+1} = A^{n+2} - B^{n+2}$, then

- (a) $I - A$ is singular
(b) $I - B$ is singular
(c) $A = B$
(d) $(I - A)(I - B)$ is non-singular

43. Let $f_1(x) = x + a$, $f_2(x) = x^2 + bx + c$ and

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ f_1(x_1) & f_1(x_2) & f_1(x_3) \\ f_2(x_1) & f_2(x_2) & f_2(x_3) \end{vmatrix}, \text{ then}$$

- (a) Δ is independent of a
(b) Δ is independent of b and c
(c) Δ is independent of x_1, x_2, x_3
(d) none of the above

44. Let $0 < \theta < \pi/2$ and

$$\Delta(x, \theta) = \begin{vmatrix} x & \tan \theta & \cot \theta \\ -\tan \theta & -x & 1 \\ \cot \theta & 1 & x \end{vmatrix}$$

then

- (a) $\Delta(0, \theta) = 0$
(b) $\Delta(x, \pi/4) = x^2 + 1$
(c) $\min_{0 < \theta < \pi/2} \Delta(1, \theta) = 2$
(d) $\Delta(x, \theta)$ is independent of x

45. Let A, B and C be 2×2 matrices with entries from the set of real numbers. Define $*$ as follows :

$$A * B = \frac{1}{2} (AB' + A'B)$$

- (a) $A * B = B * A$
(b) $A * A = A^2$
(c) $A * (B + C) = A * B + A * C$
(d) $A * I = A + A'$

46. If $f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4 \sin 2x \\ \sin^2 x & 1 + \cos^2 x & 4 \sin 2x \\ \sin^2 x & \cos^2 x & 1 + 4 \sin 2x \end{vmatrix}$,

$f(a)$ and $f(b)$ be the least and greatest value of $f(x)$, then

- (a) $f(a) = 2, f(b) = 6$ (b) $f(a) = -2, f(b) = 6$
(c) $f(a) = 2, f(b) = -6$ (d) period of $f(x)$ is π

47. Eliminating a, b, c from $x = \frac{a}{b-c}, y = \frac{b}{c-a}, z = \frac{c}{a-b}$, we get

(a) $\begin{vmatrix} 1 & -x & x \\ -y & 1 & y \\ 1 & -z & z \end{vmatrix} = 0$ (b) $\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix} = 0$

(c) $\begin{vmatrix} 1 & -x & x \\ y & 1 & -y \\ -z & z & 1 \end{vmatrix} = 0$ (d) none of these

Numerical Value Type Questions

48. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$,

then x^2 is equal to

49. The value of the determinant $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$ is

50. If $x \neq 0, y \neq 0, z \neq 0$ and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$, then

$-(x^{-1} + y^{-1} + z^{-1})$ is equal to

51. If $f(x) = \begin{vmatrix} x & \sin x & \cos x \\ x^2 & \tan x & -x^3 \\ 2x & \sin 2x & 5x \end{vmatrix}$, then $\lim_{x \rightarrow 0} \frac{f'(x)}{x}$ is



Assertion & Reason

- (A) If both assertion and reason are correct and reason is the correct explanation of assertion.
 (B) If both assertion and reason are true but reason is not the correct explanation of assertion.
 (C) If assertion is true but reason is false.
 (D) If assertion is false but reason is true.

52. Assertion : If $\Delta(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$, then

$$\Delta'(x) \neq \begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$$

Reason : $\frac{d}{dx} \{f(x)g(x)\} \neq \frac{d}{dx} f(x) \frac{d}{dx} g(x)$

- (a) A (b) B
 (c) C (d) D

53. Assertion : The system of equations possess a non trivial solution over the set of rationals $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$, then the value of k is $31/2$,

Reason : For non trivial solution $\Delta = 0$.

- (a) A (b) B
 (c) C (d) D

54. Let $A(\theta) = \begin{pmatrix} \cos \theta + \sin \theta & \sqrt{2} \sin \theta \\ -\sqrt{2} \sin \theta & \cos \theta - \sin \theta \end{pmatrix}$

Assertion : $A(\pi/3)^3 = -I$

Reason : $A(\theta)A(\phi) = A(\theta + \phi)$

- (a) A (b) B
 (c) C (d) D

55. Assertion : $\begin{vmatrix} a^2 + x^2 & ab - cx & ac + bx \\ ab + cx & b^2 + x^2 & bc - ax \\ ac - bx & bc + ax & c^2 + x^2 \end{vmatrix} = \begin{vmatrix} x & c & -b \\ -c & x & a \\ b & -a & x \end{vmatrix}^2$

Reason : $\Delta^c = \Delta^{n-1}$ where n is order of determinant, and Δ^c is the determinant of cofactors of Δ .

- (a) A (b) B
 (c) C (d) D

56. Assertion : $f(x) = \begin{vmatrix} (1+x)^{21} & (1+x)^{22} & (1+c)^{23} \\ (1+x)^{31} & (1+x)^{32} & (1+x)^{33} \\ (1+x)^{41} & (1+x)^{42} & (1+x)^{43} \end{vmatrix}$, then

coefficient of x in $f(x)$ is zero.

Reason : If $F(x) = A_0 + A_1x + A_2x^2 + \dots + A_nx^n$, then $A_1 = F'(0)$, when dash denotes the differential coefficient.

- (a) A (b) B
 (c) C (d) D

57. Assertion : $\begin{vmatrix} \cos(\theta + \alpha) & \cos(\theta + \beta) & \cos(\theta + \gamma) \\ \sin(\theta + \alpha) & \sin(\theta + \beta) & \sin(\theta + \gamma) \\ \sin(\beta - \gamma) & \sin(\gamma - \alpha) & \sin(\alpha - \beta) \end{vmatrix}$ is

independent of θ .

Reason : If $f(\theta) = c$, then $f(\theta)$ is independent of θ .

- (a) A (b) B
 (c) C (d) D

58. Suppose a, b, c are distinct real numbers.

Let $f(x) = \begin{vmatrix} x^3 + a^3 & x^2 - a^2 & x + a \\ x^3 + b^3 & x^2 - b^2 & x + b \\ x^3 + c^3 & x^2 - c^2 & x + c \end{vmatrix}$

Assertion : $f(x)$ is a polynomial of degree 3.

Reason : a, b, c are zeroes of $f(x)$.

- (a) A (b) B
 (c) C (d) D

59. Suppose $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ satisfies the equation $X^2 - 4X + 3I = O$.

Assertion : If $a + d \neq 4$, then there are just two such matrix X .

Reason : There are infinite number of matrices X , satisfying $X^2 - 4X + 3I = O$.

- (a) A (b) B
 (c) C (d) D

60. Let $a_i, b_i, c_i \in \mathbb{N}$ for $i = 1, 2, 3$ and let

$$\Delta = \begin{vmatrix} \frac{1+a_1^3b_1^3}{1+a_1b_1} & \frac{1+a_1^3b_2^3}{1+a_1b_2} & \frac{1+a_1^3b_3^3}{1+a_1b_3} \\ \frac{1+a_2^3b_1^3}{1+a_2b_1} & \frac{1+a_2^3b_2^3}{1+a_2b_2} & \frac{1+a_2^3b_3^3}{1+a_2b_3} \\ \frac{1+a_3^3b_1^3}{1+a_3b_1} & \frac{1+a_3^3b_2^3}{1+a_3b_2} & \frac{1+a_3^3b_3^3}{1+a_3b_3} \end{vmatrix}$$

Assertion : $\Delta = 0$

Reason : Δ can be written as product of two determinants.

- (a) A (b) B
 (c) C (d) D



Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching. For each question, choose the option corresponding to the correct matching.

61. Match the following List-I and List-II

Column-I

Column-II

(P) If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then the (1) 10

value of $|A^T A^{-1}|$ is

(Q) If x, y, z are cube root of unity and (2) 1

$$D = \begin{vmatrix} x^2 + y^2 & z^2 & z^2 \\ x^2 & y^2 + z^2 & x^2 \\ y^2 & y^2 & z^2 + x^2 \end{vmatrix}$$

then real part of ID is

(R) If any triangle the area $A_1 \leq \frac{b^2 + c^2}{\lambda}$, (3) 4

then largest possible numerical quantity λ is

(S) The equation $x^4 - 4x + c$ has no real (4) 0 roots, then minimum integral value of c^2 can be

Codes :

	P	Q	R	S
(A)	2	4	1	3
(B)	1	2	3	4
(C)	4	3	2	1
(D)	2	4	3	1

62. Match the following List-I and List-II

Column-I

Column-II

(A) If 2 is the root of the equation (P) e
 $|A - xI| = 0$, (where A is a non singular

matrix), $\frac{|A|}{2}$ a root of $|B - xI| = 0$,

then B can be

(B) If $e^{i\alpha}$ is the root of $|A - yI| = 0$ then (Q) $\text{adj}(A)$
 a root of $|\bar{A}^{-1} - xI| = 0$ is (where A is
 a non singular matrix)

(C) Let A_{ij} be a 2×2 non singular matrix (R) $\cos \alpha - i \sin \alpha$
 where $i, j \in \mathbb{N}$ and

$$B = \begin{vmatrix} |A_{11}| & |A_{12}| & \dots & |A_{1n}| \\ 0 & |A_{22}| & \dots & |A_{2n}| \\ 0 & 0 & |A_{33}| & \dots & |A_{3n}| \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & |A_{nn}| \end{vmatrix}$$

then $|B - \lambda I| = 0$ has root as

(D) Consider a matrix such that $\bar{A}' = A$ (S) $|A_{11}|$
 then the equation $|A - xI| = 0$ can
 have root as (where A is a non
 singular matrix)

The correct matching is

- (a) A-Q; B-R; C-S; D-P
- (b) A-R; B-Q; C-S; D-P
- (c) A-Q; B-S; C-R; D-P
- (d) A-Q; B-R; C-P; D-S



Paragraph Type Questions

Use the following passage, solve Q. 63 to Q. 67

Passage

Let $\Delta = 0$ and Δ^c denotes the determinant of cofactors, then $\Delta^c = \Delta^{n-1}$, where $n (> 0)$ is the order of Δ .

On the basis of above information, answer the following questions :

63. If a, b, c are the roots of the equation $x^3 - px^2 + r = 0$, then the value of

$$\begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix} \text{ is}$$

- (a) p^2 (b) p^4
(c) p^6 (d) p^9

64. If $l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3$ are real quantities satisfying the six relations :

$$\ell_1^2 + m_1^2 + n_1^2 = \ell_2^2 + m_2^2 + n_2^2 = \ell_3^2 + m_3^2 + n_3^2 = 1$$

$$l_2 l_3 + m_2 m_3 + n_2 n_3 = l_3 l_1 + m_3 m_1 + n_3 n_1 =$$

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0, \text{ then the value of}$$

$$\begin{vmatrix} \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \\ \ell_3 & m_3 & n_3 \end{vmatrix} \text{ is}$$

- (a) 0 (b) ± 1
(c) ± 2 (d) ± 3

65. If a, b, c are the roots of the equation $x^3 - 3x^2 + 3x + 7 = 0$, then the value of

$$\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} \text{ is}$$

- (a) 9 (b) 27
(c) 81 (d) 0

66. If $a^2 + b^2 + c^2 = \lambda^2$, then the value of

$$\begin{vmatrix} a^2 + \lambda^2 & ab + c\lambda & ca - b\lambda \\ ab - c\lambda & b^2 + \lambda^2 & bc + a\lambda \\ ac + b\lambda & bc - a\lambda & c^2 + \lambda^2 \end{vmatrix} \times \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix} \text{ is}$$

- (a) $8\lambda^6$ (b) $27\lambda^9$
(c) $8\lambda^9$ (d) $27\lambda^6$

67. Suppose $a, b, c \in \mathbb{R}$, $a + b + c > 0$, $A = bc - a^2$, $B = ca - b^2$ and $C = ab - c^2$ and

$$\begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B \end{vmatrix} = 49, \text{ then } \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \text{ equals}$$

- (a) -7 (b) 7
(c) -2401 (d) 2401

Use the following passage, solve Q. 68 to Q. 70

Passage

Elementary Transformation of a matrix :

The following operation on a matrix are called elementary operations (transformations)

1. The interchange of any two rows (or columns)
2. The multiplication of the elements of any row (or column) by any non zero number

The addition to the elements of any row (or column) the corresponding elements of any other row (or column) the corresponding elements of any other row (or column) multiplied by any number

Echelon form of matrix :

A matrix A is said to be in echelon form if

- (i) every row of A which has all its elements 0, occurs below row, which has a non-zero elements
- (ii) The first non zero element in each non-zero row is 1.
- (iii) the number of zeros before the first non zero elements in a row is less than the number of such zeroes in the next row.

[A row of matrix is said to be a zero row if all its elements are zero]

Note : Rank of a matrix does not change by application of any elementary operations.



For example $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 3 & 6 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ are echelon forms

The number of non-zero rows in the echelon form of a matrix is defined as its RANK.

For example we can reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$ into

echelon form using following elementary row transformation.

$$(i) \quad R_2 \rightarrow R_2 - 2R_1 \text{ and } R_3 \rightarrow R_3 - 3R_1 \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(ii) \quad R_2 \rightarrow R_2 - 2R_1 \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

This is the echelon form of matrix A

Number of non zero rows in the echelon form = 2

\Rightarrow Rank of the matrix A is 2

68. Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ is

- (a) 1 (b) 2
(c) 3 (d) 0

69. Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$ is

- (a) 1 (b) 2
(c) 3 (d) 4

70. The echelon form of the matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ is

$$(a) \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (b) \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$(c) \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (d) \begin{bmatrix} 1 & 3 & 4 & -\frac{3}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0 \text{ in the interval } -\frac{\pi}{4} \leq x \leq \frac{\pi}{4} \text{ is}$$

(2001)

- (a) 0 (b) 2
(c) 1 (d) 3

2. The number of values of k for which the system of equations

$$(k+1)x + 8y = 4k$$

$$kx + (k+3)y = 3k-1$$

has infinitely many solution, is

(2002)

- (a) 0 (b) 1
(c) 2 (d) infinite

3. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which

$$A^2 = B, \text{ is}$$

(2003)

- (a) 1 (b) -1
(c) 4 (d) no real values

4. If the system of equations $x + ay = 0$, $az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is

(2003)

- (a) -1 (b) 1
(c) 0 (d) no real values

5. Given $2x - y - z = 2$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ then the value of λ such that the given system of equation has no solution, is

(2004)

- (a) 3 (b) -2
(c) 0 (d) -3

6. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$ then the value of α is

(2004)

- (a) ± 1 (b) ± 2
(c) ± 3 (d) ± 5

7. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $6A^{-1} = A^2 + cA + dI$, then (c, d) is

(2005)

- (a) $(-11, 6)$ (b) $(-6, 11)$
(c) $(6, 11)$ (d) $(11, 6)$

8. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then

$P^T(Q^{2005})P$ is equal to

(2005)

- (a) $\begin{bmatrix} 1 & \frac{\sqrt{3}}{2} \\ 0 & 2005 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & 2005 \\ 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & 2005 \\ \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$

9. The number of 3×3 matrices A whose entries are either

0 or 1 and for which the system $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly

two distinct solutions, is

(2010)

- (a) 0 (b) $2^9 - 1$
(c) 168 (d) 2



10. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-

singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of

a, b and c is either ω or ω^2 . Then, the number of distinct matrices in the set S is (2011)

- (a) 2 (b) 6
(c) 4 (d) 8

11. How many 3×3 matrices M with entries from $\{0, 1, 2\}$ are there, for which the sum of the diagonal entries of $M^T M$ is 5? (2017)

- (a) 126 (b) 198
(c) 162 (d) 135

12. Let $M = \begin{bmatrix} \sin^4 \theta & -1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{bmatrix} = \alpha I + \beta M^{-1}$, where

$\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers and I is the 2×2 identity matrix.

If α^* is the minimum of the set $\{\alpha(\theta) : \theta \in [0, 2\pi)\}$ and

β^* is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi)\}$, then

the value of $\alpha^* + \beta^*$ is (2019)

- (a) $\frac{-37}{16}$ (b) $\frac{-29}{16}$
(c) $\frac{-31}{16}$ (d) $\frac{-17}{16}$

Objective Questions II

[One or more than one correct option]

13. Let M and N be two 3×3 non-singular skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P, then $M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$ is equal to (2011)

- (a) M^2 (b) $-N^2$
(c) $-M^2$ (d) MN

14. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is/are (2012)

- (a) -2 (b) -1
(c) 1 (d) 2

15. For 3×3 matrices M and N, which of the following statement(s) is (are) not correct? (2013)

- (a) $N^T M N$ is symmetric or skew-symmetric, according as M is symmetric or skew-symmetric
(b) $MN - NM$ is skew symmetric for all symmetric matrices M and N
(c) MN is symmetric for all symmetric matrices M and N
(d) $(\text{adj } M)(\text{adj } N) = \text{adj } (MN)$ for all invertible matrices M and N

16. Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then (2014)

- (a) determinant of $(M^2 + MN^2)$ is 0
(b) there is a 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix
(c) determinant of $(M^2 + MN^2) \geq 1$
(d) for a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

17. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if (2014)

- (a) the first column of M is the transpose of the second row of M
(b) the second row of M is the transpose of the first column of M
(c) M is a diagonal matrix with nonzero entries in the main diagonal
(d) the product of entries in the main diagonal of M is not the square of an integer

18. Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha \quad (2015)$$

- (a) -4 (b) 9
(c) -9 (d) 4



19. Let X and Y be two arbitrary, 3×3 , non-zero, skew-symmetric matrices and Z be an arbitrary 3×3 , non zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric ? (2015)

- (a) $Y^3Z^4 - Z^4Y^3$ (b) $X^{44} + Y^{44}$
(c) $X^4Z^3 - Z^3X^4$ (d) $X^{23} - Y^{23}$

20. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = [q_{ij}]$

is a matrix such that $PQ = kI$, where $k \in \mathbb{R}$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and \det

$(Q) = \frac{k^2}{2}$, then (2016)

- (a) $\alpha = 0, k = 8$ (b) $4\alpha - k + 8 = 0$
(c) $\det(P \operatorname{adj}(Q)) = 2^9$ (d) $\det(Q \operatorname{adj}(P)) = 2^{13}$

21. Which of the following is(are) **NOT** the square of a 3×3 matrix with real entries? (2017)

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

22. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that

$b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables)

$$-x + 2y + 5z = b_1$$

$$2x - 4y + 3z = b_2$$

$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one

solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$? (2018)

- (a) $x + 2y + 3z = b_1, 4y + 5z = b_2$ and $x + 2y + 6z = b_3$
(b) $x + y + 3z = b_1, 5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$
(c) $-x + 2y - 5z = b_1, 2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$
(d) $x + 2y + 5z = b_1, 2x + 3z = b_2$ and $x + 4y - 5z = b_3$

23. Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and $\operatorname{adj} M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ Where a

and b are real numbers. Which of the following options is /are correct? (2019)

- (a) $\det(\operatorname{adj} M^2) = 81$
(b) $a + b = 3$

(c) If $M \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

(d) $(\operatorname{adj} M)^{-1} + \operatorname{adj} M^{-1} = -M$

24. Let $x \in \mathbb{R}$ and let

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \text{ and } R = PQP^{-1}$$

Then which of the following is/are correct (2019)

- (a) There exists a real number x such that $PQ = QP$

(b) $\det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$ for all $x \in \mathbb{R}$

- (c) For $x = 1$ there exists a unit vector $\alpha \hat{i} + \beta \hat{j} + \gamma \hat{k}$ for

which are $R \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(d) For $x = 0$ if $R \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6 \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$ then $a + b = 5$



25. Let

$$P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\text{and } X = \sum_{k=1}^6 P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T.$$

Where P_k^T denotes the transpose of matrix P_k . Then which of the following options is/are correct? (2019)

- (a) X is a symmetric matrix
(b) The sum of diagonal entries of X is 18.

(c) if $X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha=30$

(d) $X - 30I$ is an invertible matrix

26. Let M be a 3×3 invertible matrix with real entries and let I denote the 3×3 identity matrix. If $M^{-1} = \text{adj}(\text{adj } M)$, then which of the following statement is/are ALWAYS TRUE? (2020)

- (a) $M = I$ (b) $\det M = 1$
(c) $M^2 = I$ (d) $(\text{adj } M)^2 = I$

27. For any 3×3 matrix M, let $|M|$ denote the determinant of M.

$$\text{Let } E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\text{and } F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a non-singular matrix of order 3×3 , then which of

the following statements is/are TRUE?

(2021)

(a) $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

(c) $|(EF)^3| > |EF|^2$

(d) Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

28. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that $(I - EF)$ is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is (are) TRUE? (2021)

(a) $|FE| = |I - FE||FGE|$

(b) $(I - FE)(I + FGE) = I$

(c) $EFG = GEF$

(d) $(I - FE)(I - FGE) = I$

Numerical Value Type Questions

29. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $A^T A = I$, then find value of $a^3 + b^3 + c^3$. (2003)

30. Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If $\det(\text{adj } A) + \det(\text{adj } B) = 10^6$, then $[k]$ is equal to....

(2010)



31. The number of all possible values of θ , where $0 < \theta < \pi$, for which the system of equations

$$(y+z) \cos 3\theta = (xyz) \sin 3\theta$$

$$x \sin 3\theta = \frac{2 \cos 3\theta}{y} + \frac{2 \sin 3\theta}{z}$$

and $(xyz) \sin 3\theta = (y+2z) \cos 3\theta + y \sin 3\theta$ have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is (2010)

32. Let ω be the complex number $\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$. Then the number of distinct complex number z satisfying

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \text{ is equal to} \quad (2010)$$

33. Let M be a 3×3 matrix satisfying

$$M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \text{ and } M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

Then, the sum of the diagonal entries of M is (2011)

34. Let $z = \frac{-1 + \sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s \in \{1, 2, 3\}$.

$$\text{Let } P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix} \text{ and } I \text{ be the identity matrix of}$$

order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is (2016)

35. The total number of distinct $x \in \mathbb{R}$ for which

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10 \text{ is} \quad (2016)$$

36. For a real number α , if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ -z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \text{ of linear equations, has}$$

infinitely many solutions, then $1 + \alpha + \alpha^2 =$ (2017)

37. The trace of a square matrix is defined to be the sum of its diagonal entries. If A is a 2×2 matrix such that the trace of A is 3 and the trace of A^3 is -18, then the value of the determinant of A is _____. (2020)

38. Let α, β and γ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$

$$4x + 5y + 6z = \beta$$

$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let $|M|$ represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

The value of $|M|$ is _____. (2021)

Assertion & Reason

- (A) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.
(B) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.
(C) If ASSERTION is true, REASON is false
(D) If ASSERTION is false, REASON is true

39. Consider the system of equations

$$x - 2y + 3z = -1, -x + y - 2z = k, x - 3y + 4z = 1$$

Assertion : The system of equations has no solution for $k \neq 3$.

Reason : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$

(1997)

- (a) A (b) B
(c) C (d) D



Paragraph Type Questions

Using the following passage, solve Q.40 to Q.42

Passage

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}, \text{ If } U_1, U_2 \text{ are } U_3 \text{ are columns matrices}$$

$$\text{satisfying } AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \text{ and } AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \text{ and } U \text{ is}$$

3×3 matrix when columns are U_1, U_2, U_3 then answer the following questions.

40. The value of $|U|$ is (2006)

- (a) 3 (b) -3
(c) $3/2$ (d) 2

41. The sum of the elements of U^{-1} is (2006)

- (a) -1 (b) 0
(c) 1 (d) 3

42. The value of $[3 \ 2 \ 0] U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is (2006)

- (a) 5 (b) $5/2$
(c) 4 (d) $3/2$

Using the following passage, solve Q.43 to Q.45

Passage

Let p be an odd prime number and T_p be the following set of 2×2 matrices

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}; a, b, c \in \{0, 1, 2, \dots, p-1\} \right\} \quad (2010)$$

43. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and $\det(A)$ is divisible by p is

- (a) $(p-1)^2$ (b) $2(p-1)$
(c) $(p-1)^2 + 1$ (d) $2p-1$

44. The number of A in T_p such that the trace of A is not divisible by p but $\det(A)$ is divisible by p is

[Note : The trace of a matrix is the sum of its diagonal entries]

- (a) $(p-1)(p^2-p+1)$ (b) $p^3-(p-1)^2$
(c) $(p-1)^2$ (d) $(p-1)(p^2-2)$

45. The number of A in T_p such that $\det(A)$ is not divisible by p , is

- (a) $2p^2$ (b) p^3-5p
(c) p^3-3p (d) p^3-p^2

Answer Key



CHAPTER -1 | DETERMINANTS & MATRICES

EXERCISE - 1: BASIC OBJECTIVE QUESTIONS



DIRECTION TO USE -

Scan the QR code and check detailed solutions.

- | | | | | |
|----------|---------|---------|---------|---------|
| 1. (d) | 2. (b) | 3. (d) | 4. (b) | 5. (a) |
| 6. (a) | 7. (d) | 8. (b) | 9. (d) | 10. (b) |
| 11. (d) | 12. (a) | 13. (c) | 14. (c) | 15. (b) |
| 16. (a) | 17. (c) | 18. (b) | 19. (a) | 20. (b) |
| 21. (b) | 22. (c) | 23. (a) | 24. (a) | 25. (a) |
| 26. (b) | 27. (b) | 28. (a) | 29. (c) | 30. (b) |
| 31. (d) | 32. (d) | 33. (b) | 34. (c) | 35. (d) |
| 36. (b) | 37. (d) | 38. (a) | 39. (c) | 40. (b) |
| 41. (c) | 42. (c) | 43. (c) | 44. (d) | 45. (a) |
| 46. (a) | 47. (c) | 48. (b) | 49. (b) | 50. (c) |
| 51. (c) | 52. (a) | 53. (b) | 54. (a) | 55. (d) |
| 56. (b) | 57. (b) | 58. (b) | 59. (a) | 60. (b) |
| 61. (1) | 62. (2) | 63. (0) | 64. (0) | 65. (4) |
| 66. (6) | 67. (0) | 68. (3) | 69. (3) | 70. (4) |
| 71. (16) | 72. (2) | 73. (1) | 74. (2) | 75. (2) |

EXERCISE - 2: PREVIOUS YEAR JEE MAIN QUESTIONS



DIRECTION TO USE -

Scan the QR code and check detailed solutions.

- | | | | | |
|----------------|---------------|--------------|---------------|-------------|
| 1. (c) | 2. (a) | 3. (d) | 4. (c) | 5. (c) |
| 6. (d) | 7. (d) | 8. (a) | 9. (d) | 10. (d) |
| 11. (b) | 12. (b) | 13. (b) | 14. (c) | 15. (d) |
| 16. (d) | 17. (d) | 18. (d) | 19. (a) | 20. (b) |
| 21. (c) | 22. (a) | 23. (b) | 24. (c) | 25. (b) |
| 26. (b) | 27. (d) | 28. (b) | 29. (d) | 30. (d) |
| 31. (b) | 32. (b) | 33. (c) | 34. (a) | 35. (d) |
| 36. (c) | 37. (c) | 38. (a) | 39. (8.00) | 40. (b) |
| 41. (a) | 42. (b) | 43. (b) | 44. (d) | 45. (c) |
| 46. (a) | 47. (d) | 48. (a) | 49. (d) | 50. (d) |
| 51. (c) | 52. (d) | 53. (c) | 54. (10.00) | 55. (c) |
| 56. (8.00) | 57. (d) | 58. (5.00) | 59. (a) | 60. (b) |
| 61. (b) | 62. (b) | 63. (a) | 64. (c) | 65. (b) |
| 66. (a) | 67. (b) | 68. (3.00) | 69. (c) | 70. (c) |
| 71. (a) | 72. (13.00) | 73. (d) | 74. (672.00) | |
| 75. (b) | 76. (a) | 77. (b) | 78. (c) | 79. (c) |
| 80. (b) | 81. (b) | 82. (d) | 83. (1.00) | |
| 84. (910.00) | | 85. (d) | 86. (108.00) | |
| 87. (11.00) | 88. (d) | 89. (6.00) | 90. (5.00) | 91. (b) |
| 92. (a) | 93. (2020.00) | | 94. (3125.00) | |
| 95. (b) | 96. (d) | 97. (a) | 98. (b) | 99. (c) |
| 100. (b) | 101. (a) | 102. (4.00) | 103. (c) | 104. (d) |
| 105. (5.00) | 106. (d) | 107. (c) | 108. (d) | 109. (a) |
| 110. (c) | 111. (8.00) | 112. (d) | 113. (c) | 114. (c) |
| 115. (4.00) | 116. (b) | 117. (c) | 118. (13.00) | 119. (7.00) |
| 120. (21.00) | | 121. (b) | 122. (c) | 123. (c) |
| 124. (a) | 125. (d) | 126. (d) | 127. (540.00) | |
| 128. (17.00) | | 129. (a) | 130. (c) | 131. (c) |
| 132. (d) | 133. (b) | 134. (6.00) | 135. (d) | |
| 136. (766.00) | | 137. (36.00) | | 138. (a) |
| 139. (1.00) | 140. (b) | 141. (a) | 142. (16.00) | 143. (d) |
| 144. (2020.00) | | 145. (2.00) | | |

CHAPTER -1 | DETERMINANTS & MATRICES

EXERCISE - 3 :
ADVANCED OBJECTIVE QUESTIONS

DIRECTION TO USE -

Scan the QR code and check detailed solutions.

1. (b) 2. (d) 3. (a) 4. (c) 5. (d)
 6. (b) 7. (d) 8. (d) 9. (a) 10. (a)
 11. (b) 12. (b) 13. (a) 14. (a) 15. (a)
 16. (b) 17. (a) 18. (c) 19. (c) 20. (d)
 21. (a) 22. (a) 23. (d) 24. (c) 25. (d)
 26. (d) 27. (a,b,c,d) 28. (a,c)
 29. (a,b,c,d) 30. (a,b,c) 31. (b,d) 32. (a,d)
 33. (a,c) 34. (a,b,c,d) 35. (b,c,d)
 36. (a,b,c) 37. (a,b,c) 38. (a,b,d) 39. (a,b,c)
 40. (a,b,c) 41. (a,b,c) 42. (a,b,c) 43. (a,b) 44. (a)
 45. (c) 46. (b,d) 47. (b,c) 48. (81) 49. (0)
 50. (3) 51. (4) 52. (a) 53. (d) 54. (a)
 55. (a) 56. (b) 57. (b) 58. (c) 59. (b)
 60. (d) 61. (d) 62. (a) 63. (c) 64. (b)
 65. (d) 66. (c) 67. (a) 68. (b) 69. (c)
 70. (a)

EXERCISE - 4 :
PREVIOUS YEAR JEE ADVANCED QUESTIONS

DIRECTION TO USE -

Scan the QR code and check detailed solutions.

1. (c) 2. (b) 3. (d) 4. (a) 5. (b)
 6. (c) 7. (b) 8. (b) 9. (a) 10. (a)
 11. (b) 12. (b) 13. (c) 14. (a,d) 15. (c,d)
 16. (a,b) 17. (c,d) 18. (b,c) 19. (c,d) 20. (b,c)
 21. (a,b) 22. (a,c,d) 23. (b,c,d) 24. (b,d)
 25. (a,b,c) 26. (b,c,d) 27. (a,b,d)
 28. (a,b,c) 29. (4) 30. (4) 31. (3) 32. (1)
 33. (9) 34. (1) 35. (2) 36. (1)
 37. (5.00) 38. (1.00) 39. (a) 40. (a)
 41. (b) 42. (a) 43. (d) 44. (c) 45. (d)

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2. After sharing resources with students, consider deleting outdated data if necessary.
3. It's a NO Nuisance groups, single nuisance and you will be removed.
 - No introductions.
 - No greetings or wish messages.
 - No personal chats or messages.
 - No spam. Or voice calls
 - Share and seek learning resources only.

Please only share and request learning resources. For assistance, contact the helpline via WhatsApp: +91-95208-77777.

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Team

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SKILL MODULES BEING OFFERED IN MIDDLE SCHOOL



Artificial Intelligence



Beauty & Wellness



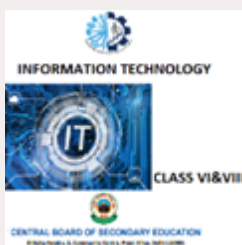
Design Thinking & Innovation



Financial Literacy



Handicrafts



Information Technology



Marketing/Commercial Application



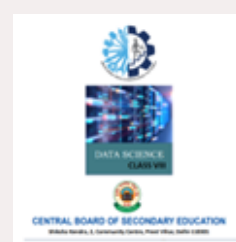
Mass Media - Being Media Literate



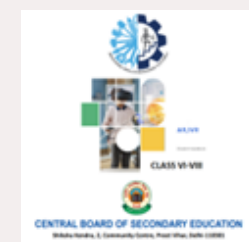
Travel & Tourism



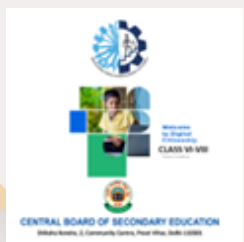
Coding



Data Science (Class VIII only)



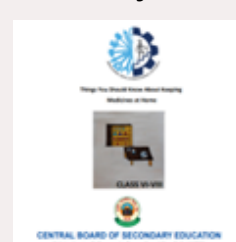
Augmented Reality / Virtual Reality



Digital Citizenship



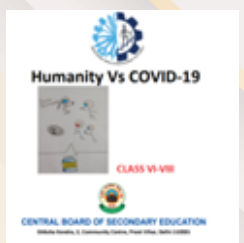
Life Cycle of Medicine & Vaccine



Things you should know about keeping Medicines at home



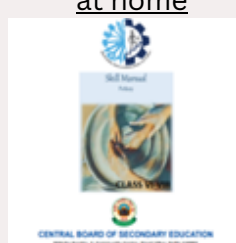
What to do when Doctor is not around



Humanity & Covid-19



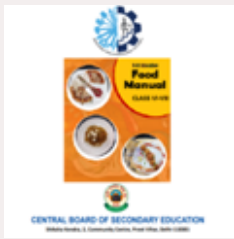
Blue Pottery



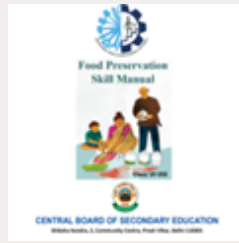
Pottery



Block Printing



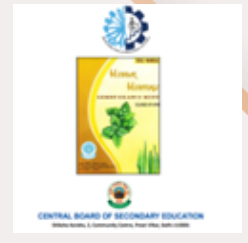
Food



Food Preservation



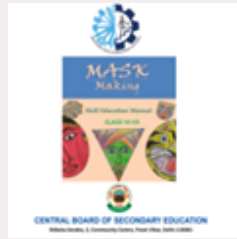
Baking



Herbal Heritage



Khadi



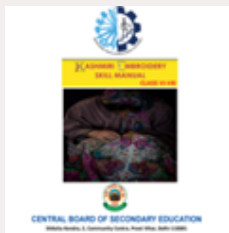
Mask Making



Mass Media



Making of a Graphic Novel



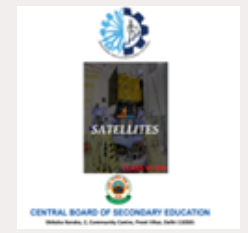
Kashmiri Embroidery



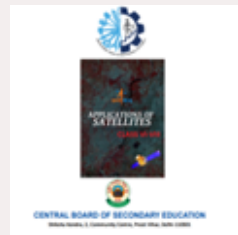
Embroidery



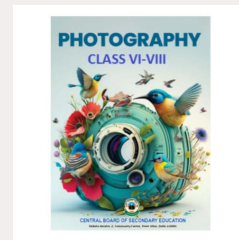
Rockets



Satellites



Application of Satellites



Photography

SKILL SUBJECTS AT SECONDARY LEVEL (CLASSES IX – X)



Retail



Information Technology



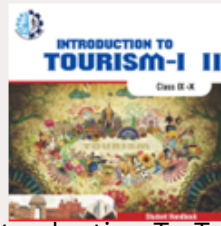
Security



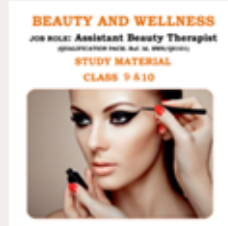
Automotive



Introduction To Financial Markets



Introduction To Tourism



Beauty & Wellness



Agriculture



Food Production



Front Office Operations



Banking & Insurance



Marketing & Sales



Health Care



Apparel



Multi Media



Multi Skill Foundation Course



Artificial Intelligence



Physical Activity Trainer



Data Science



Electronics & Hardware (NEW)

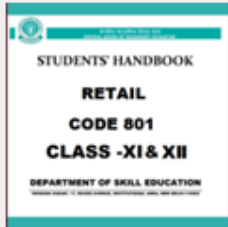


Foundation Skills For Sciences (Pharmaceutical & Biotechnology)(NEW)

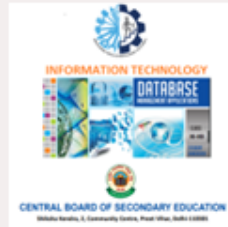


Design Thinking & Innovation (NEW)

SKILL SUBJECTS AT SR. SEC. LEVEL (CLASSES XI – XII)



Retail



Information Technology



Web Application



Automotive



Financial Markets Management



Tourism



Beauty & Wellness



Agriculture



Food Production



Front Office Operations



Banking



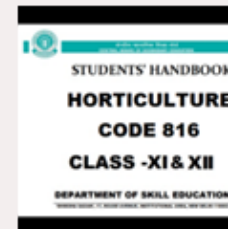
Marketing



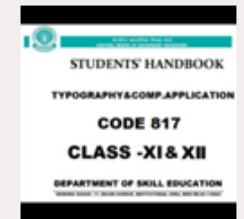
Health Care



Insurance



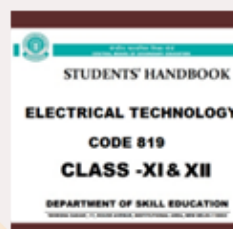
Horticulture



Typography & Comp.
Application



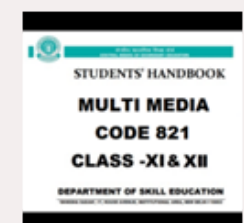
Geospatial Technology



Electrical Technology



Electronic Technology



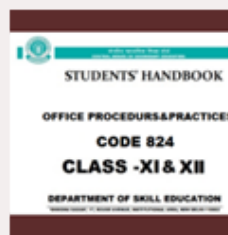
Multi-Media



Taxation



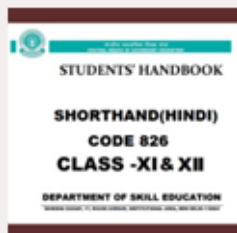
Cost Accounting



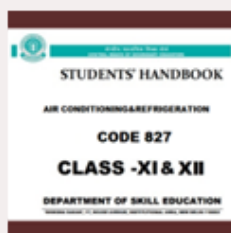
Office Procedures & Practices



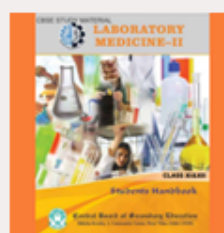
Shorthand (English)



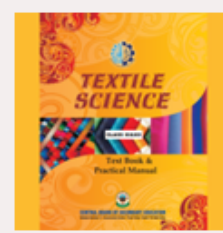
Shorthand (Hindi)



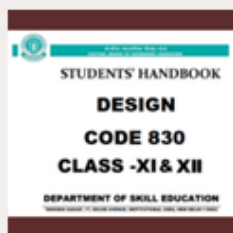
Air-Conditioning & Refrigeration



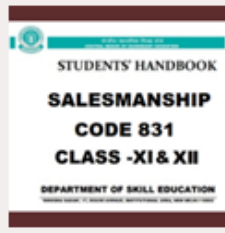
Medical Diagnostics



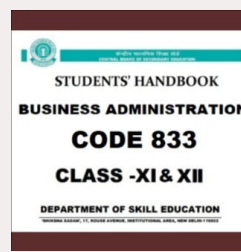
Textile Design



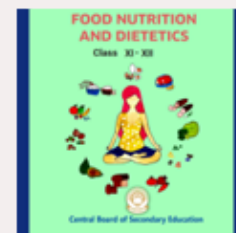
Design



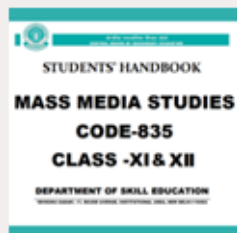
Salesmanship



Business Administration



Food Nutrition & Dietetics



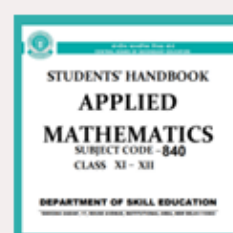
Mass Media Studies



Library & Information Science



Fashion Studies



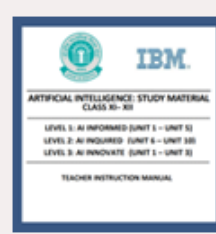
Applied Mathematics



Yoga



Early Childhood Care & Education



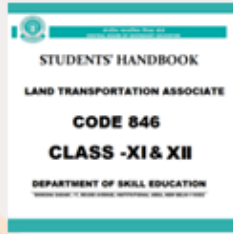
Artificial Intelligence



Data Science



Physical Activity Trainer(new)



Land Transportation Associate (NEW)



Electronics & Hardware (NEW)



Design Thinking & Innovation (NEW)

